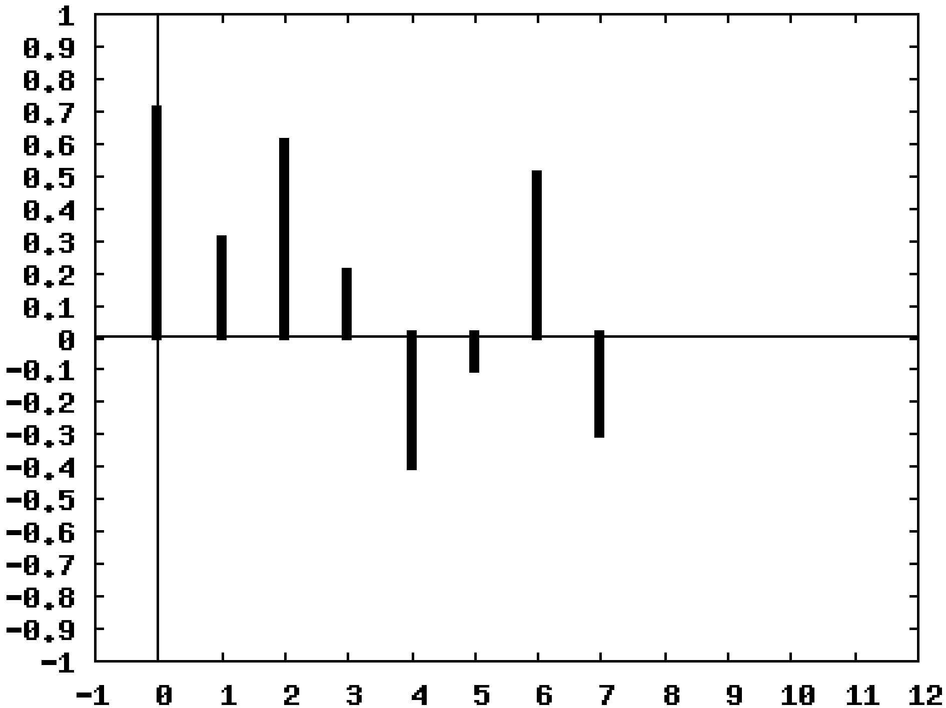
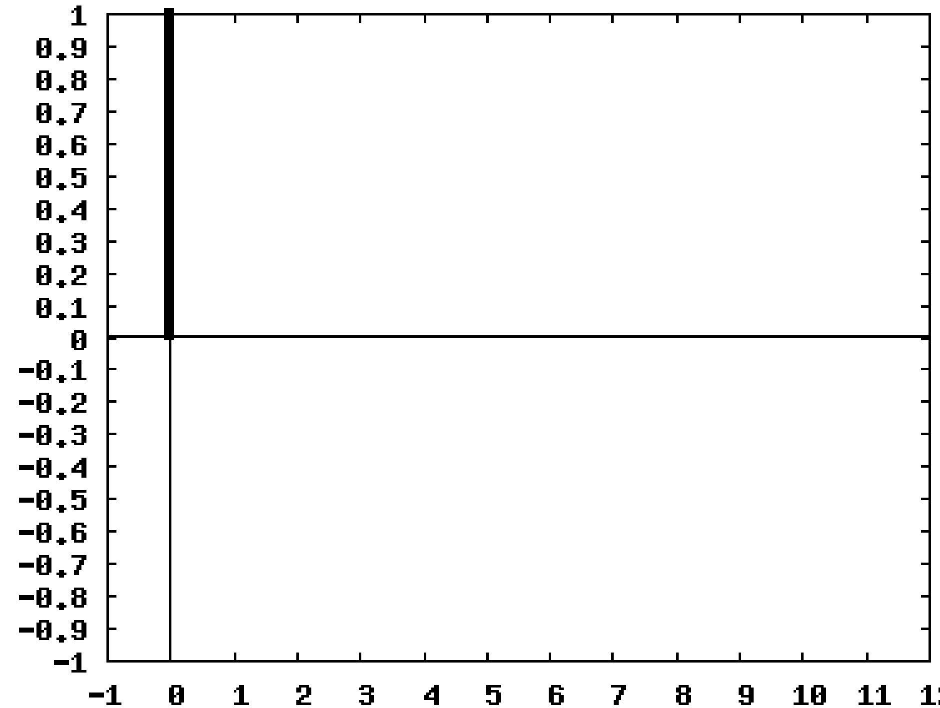
Introducción a la Teoría del Procesamiento Digital de Señales de Audio

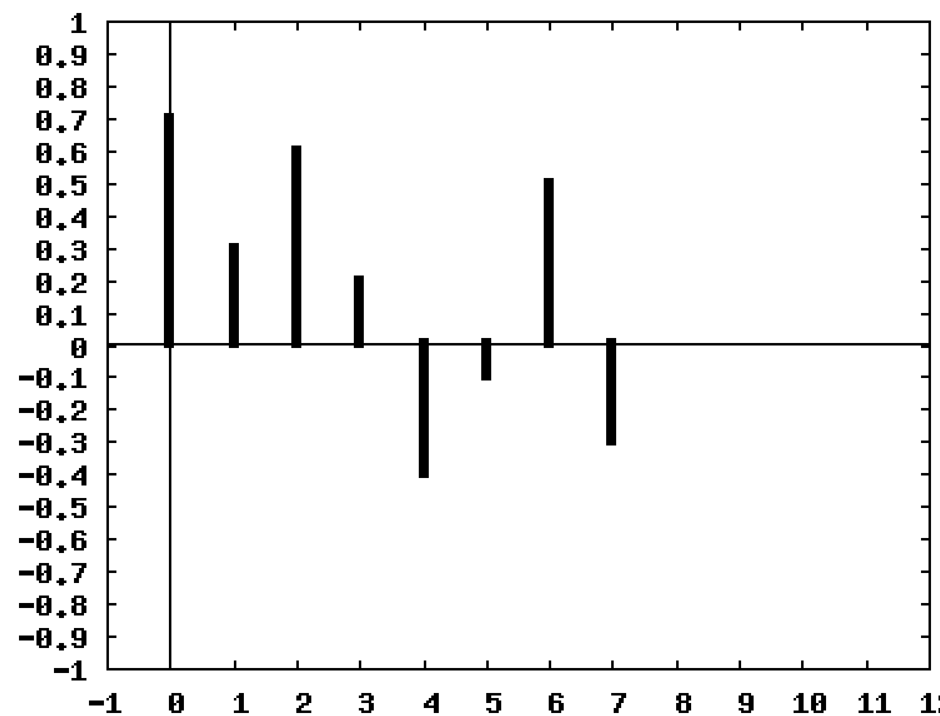
clase 4

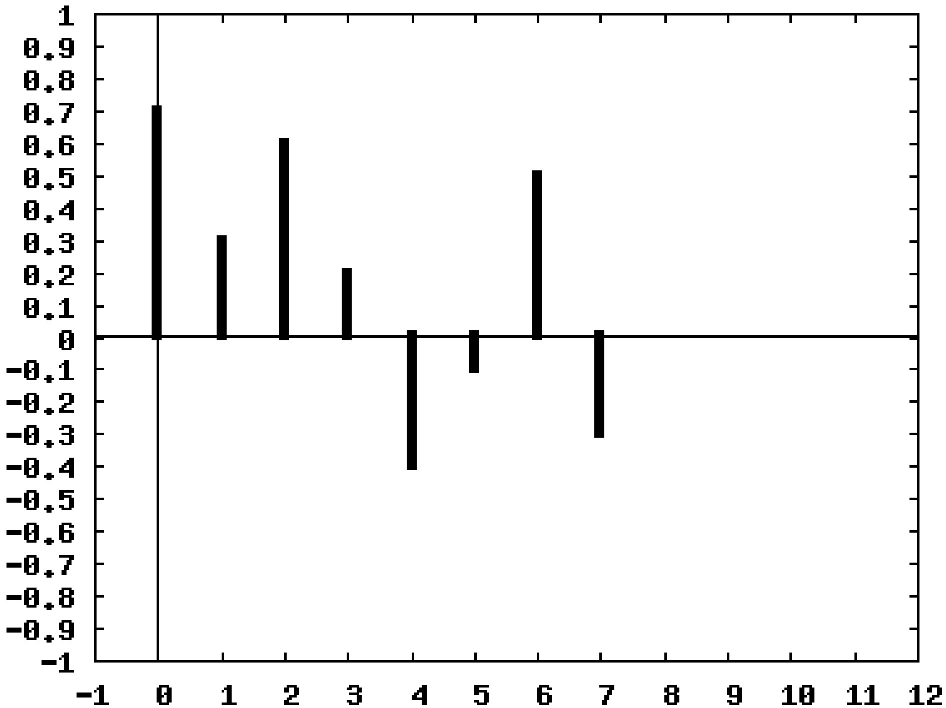
convolución

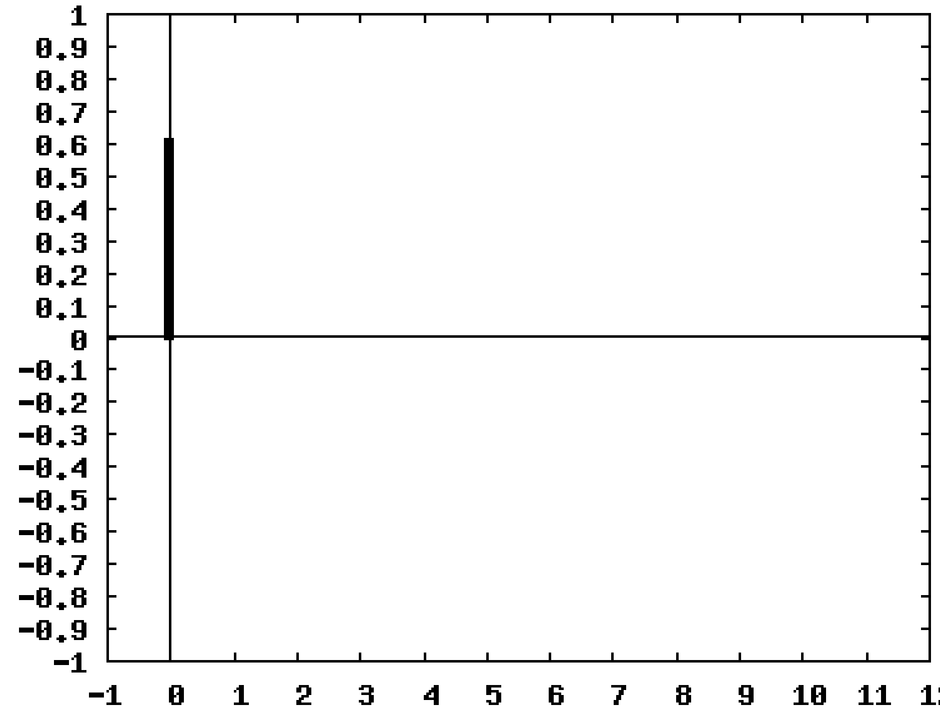
$$x[n] * h[n] = y[n]$$

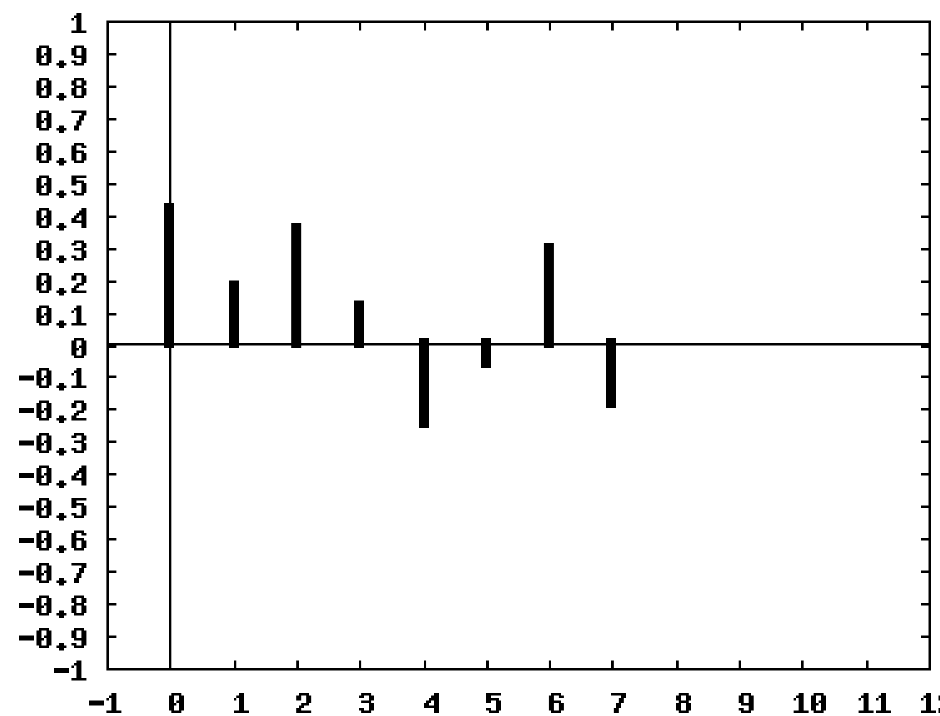

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

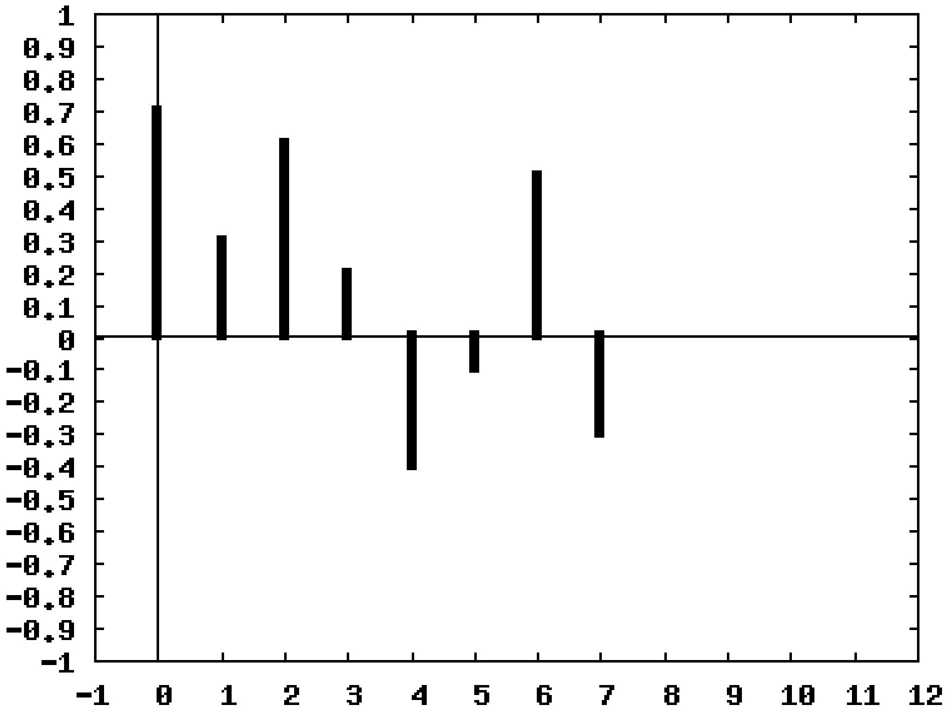

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

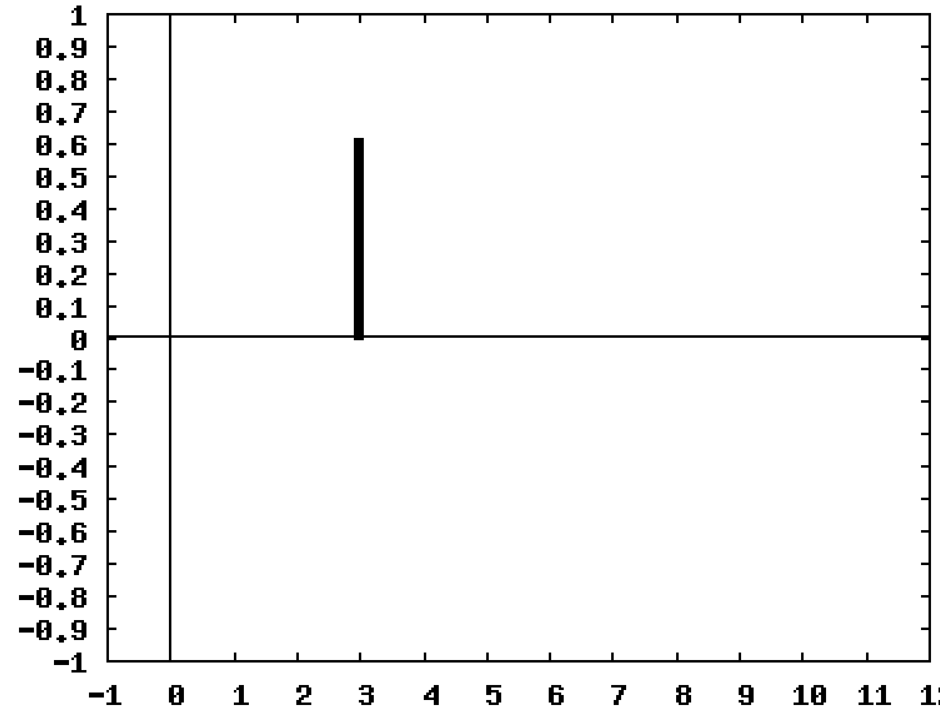

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

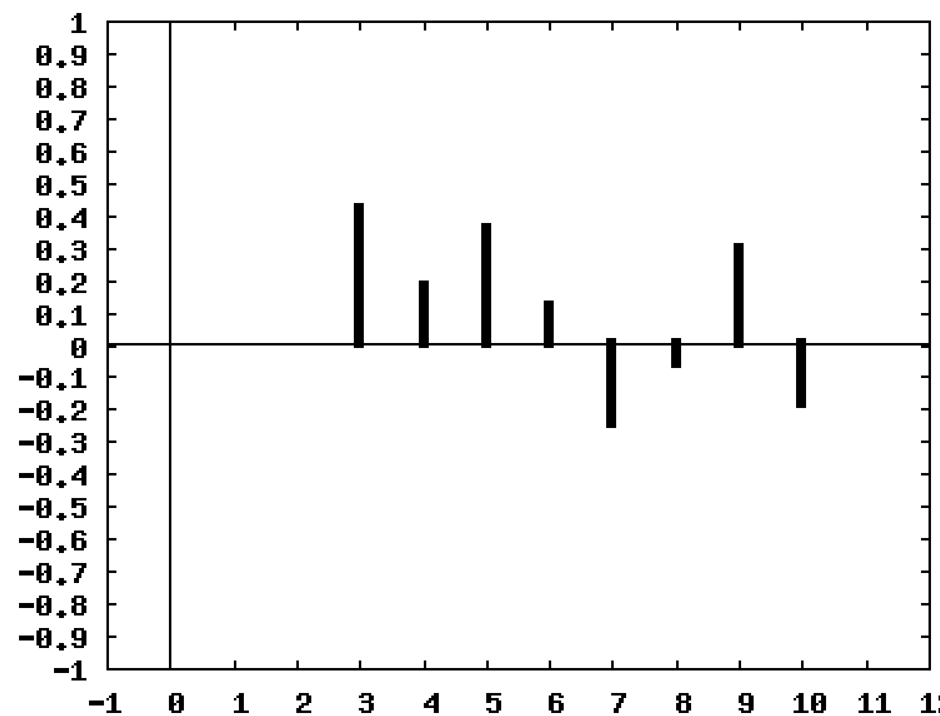

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

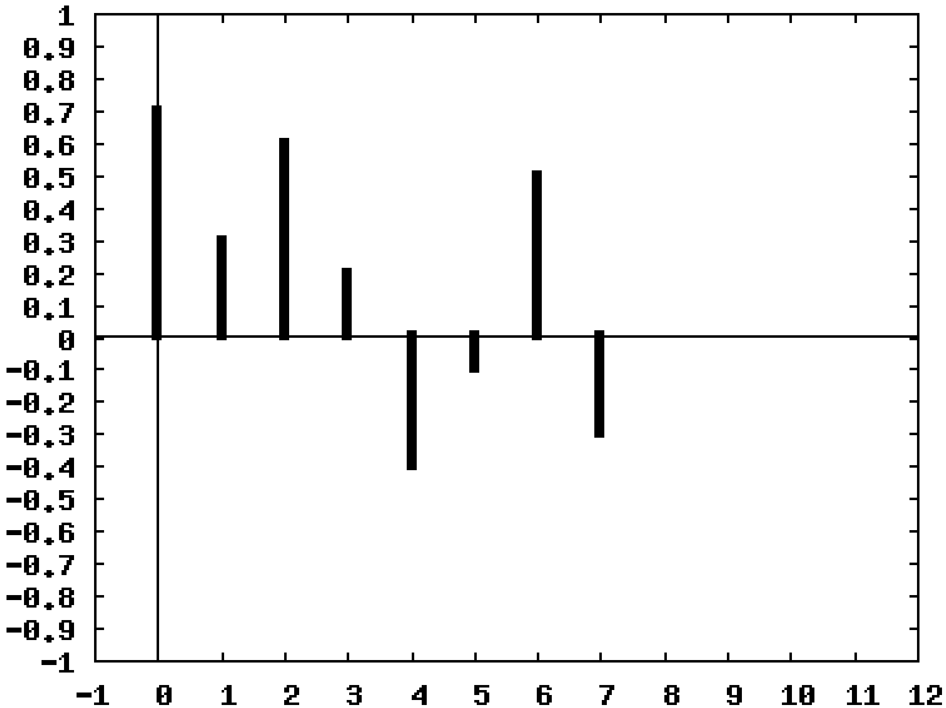

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

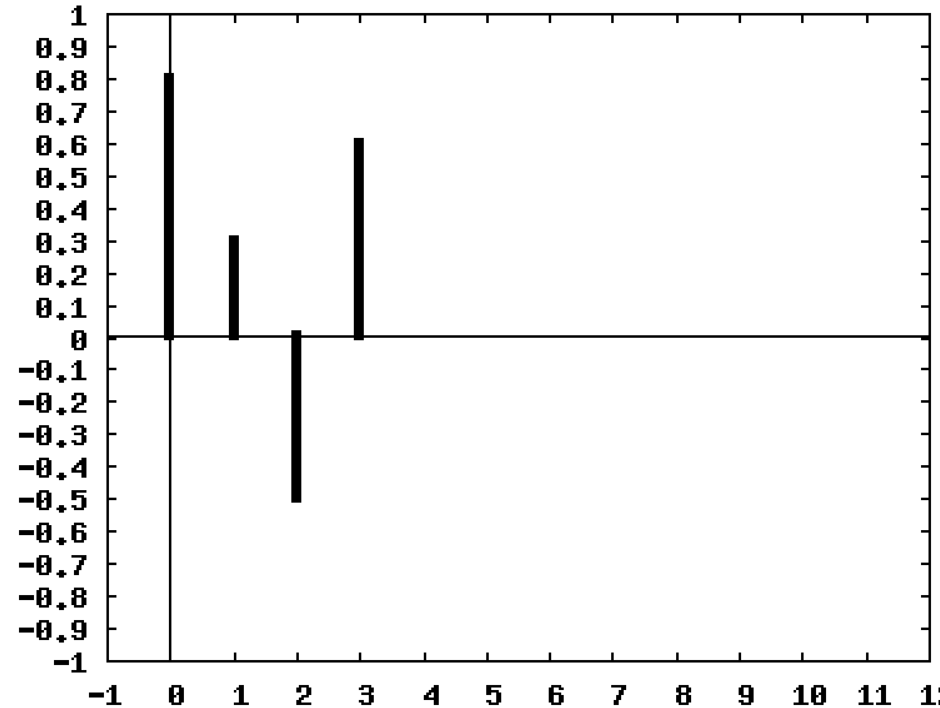

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

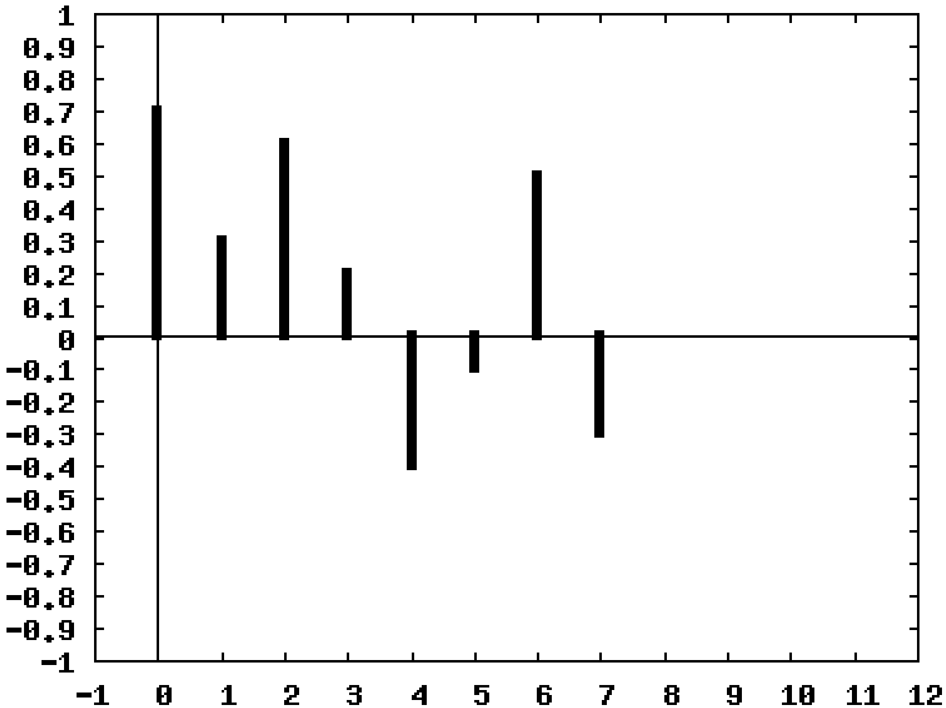

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

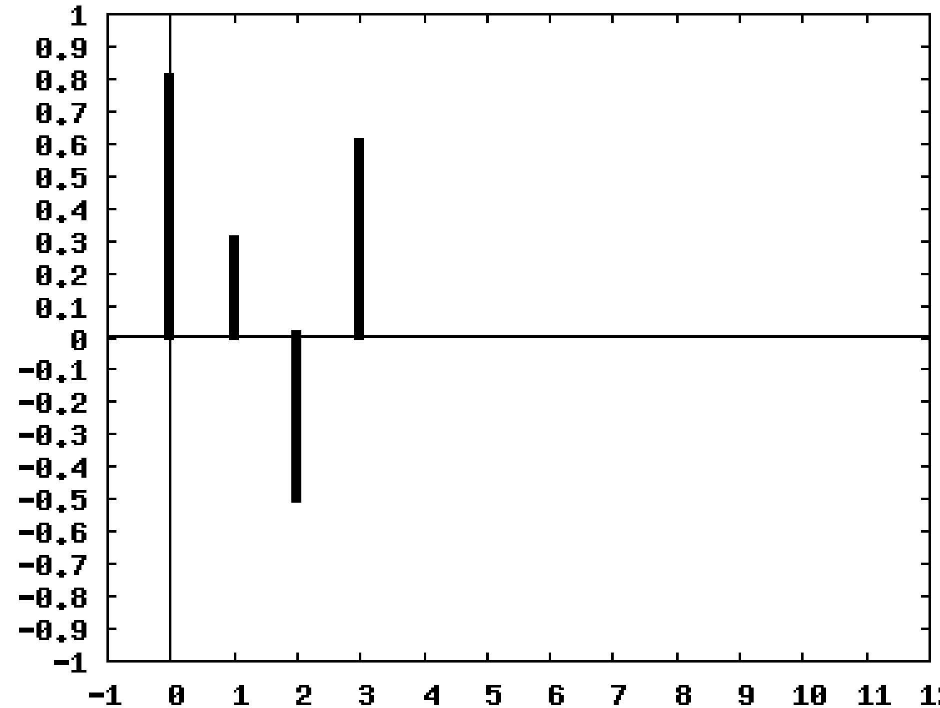
Al multiplicar la señal $x_{[n]}$ por una muestra $h_{[i]}$ perteneciente a $h_{[n]}$, le estamos aplicando dos operaciones:

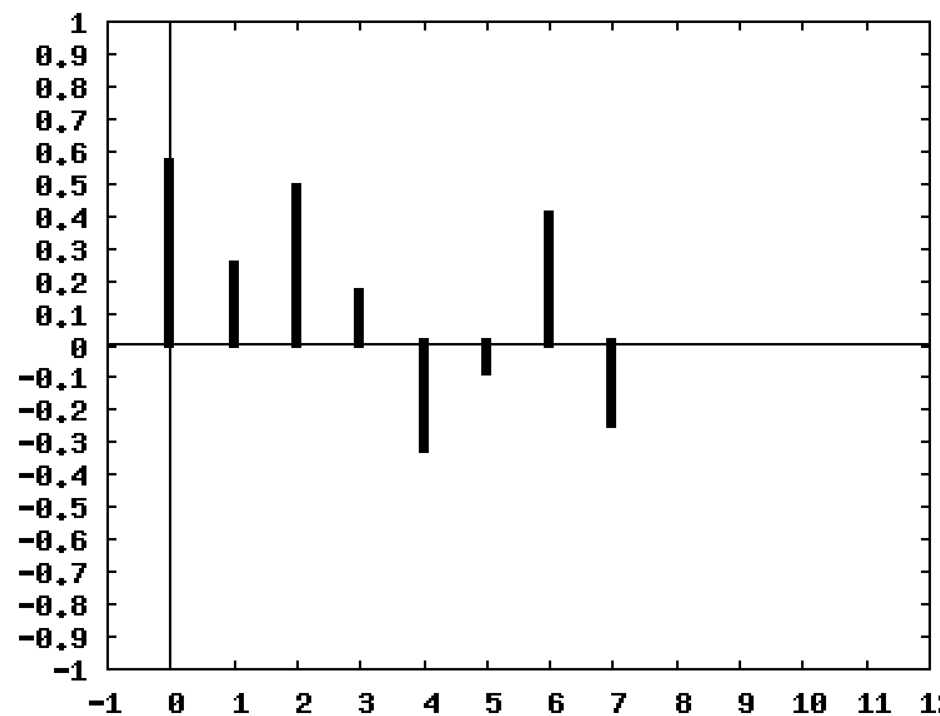
- un escalamiento proporcional a la amplitud de la muestra $h_{[i]}$
- un desplazamiento de i muestras

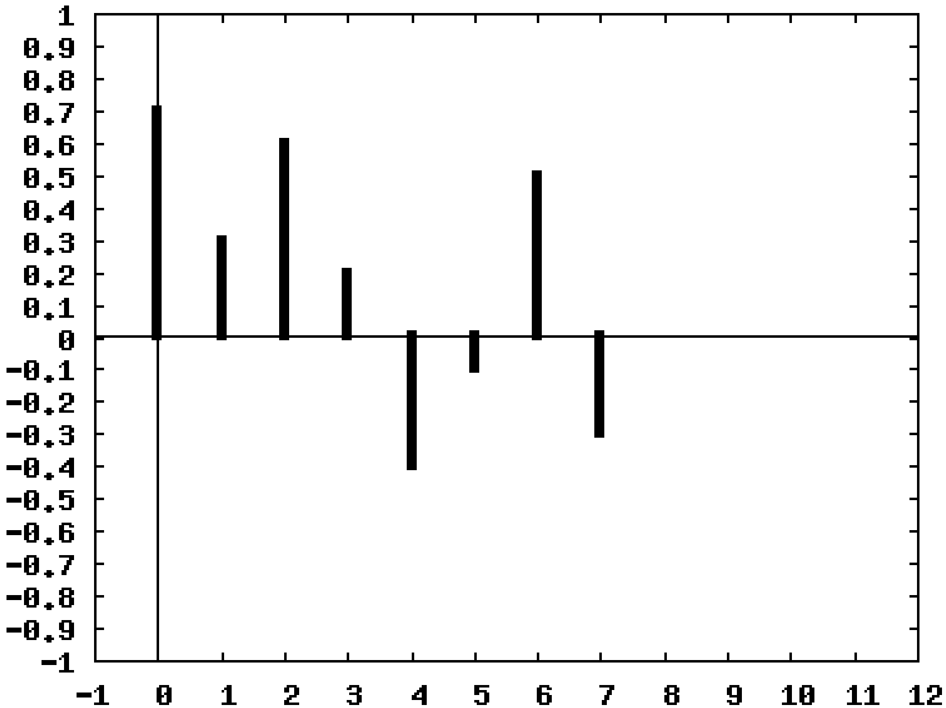

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

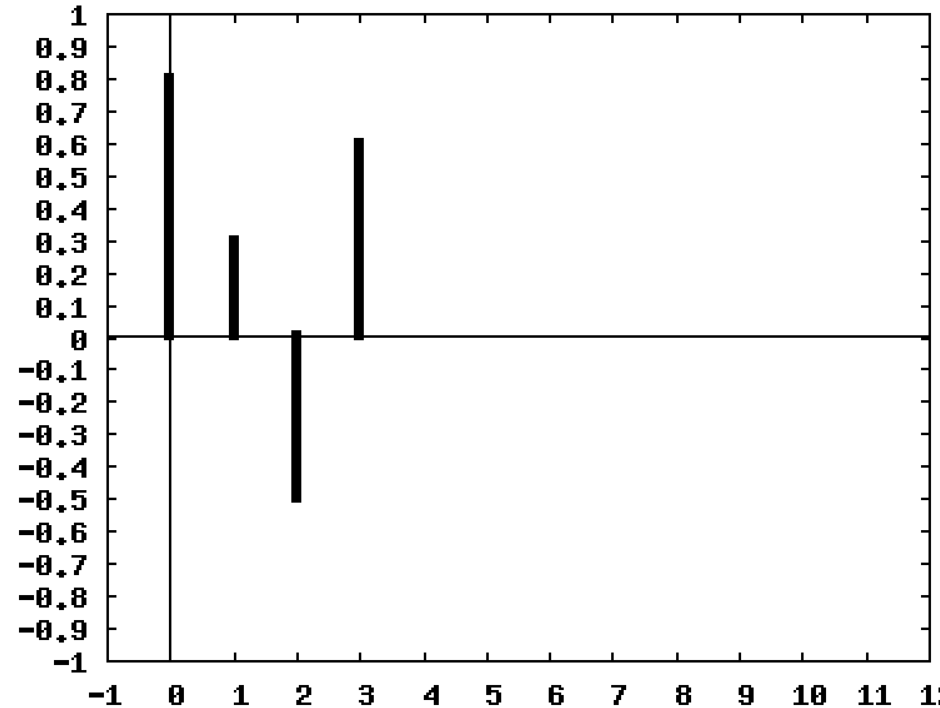

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

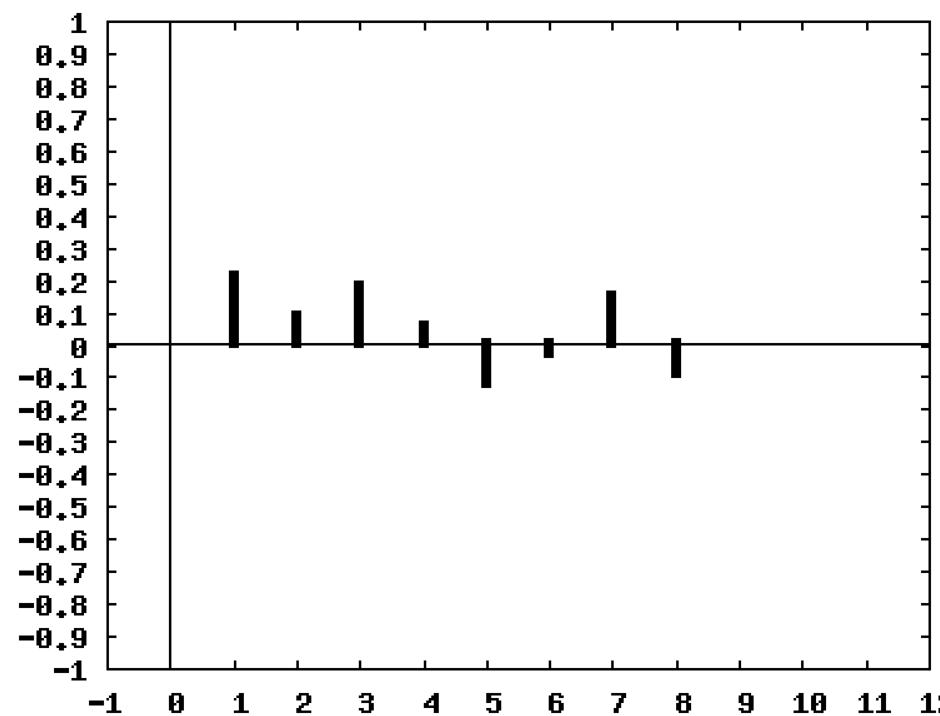

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

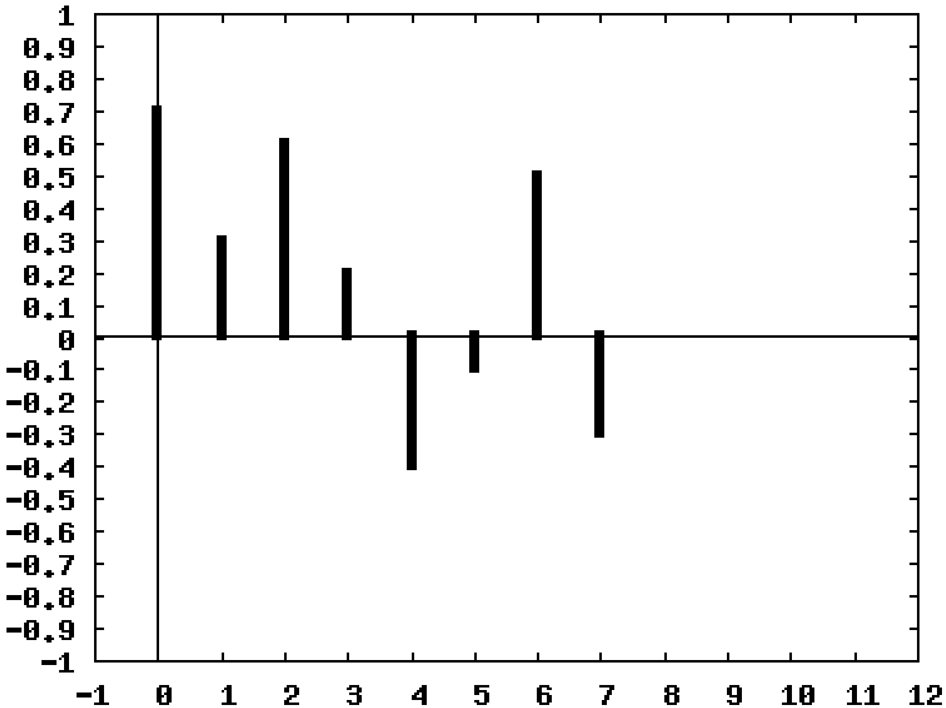

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

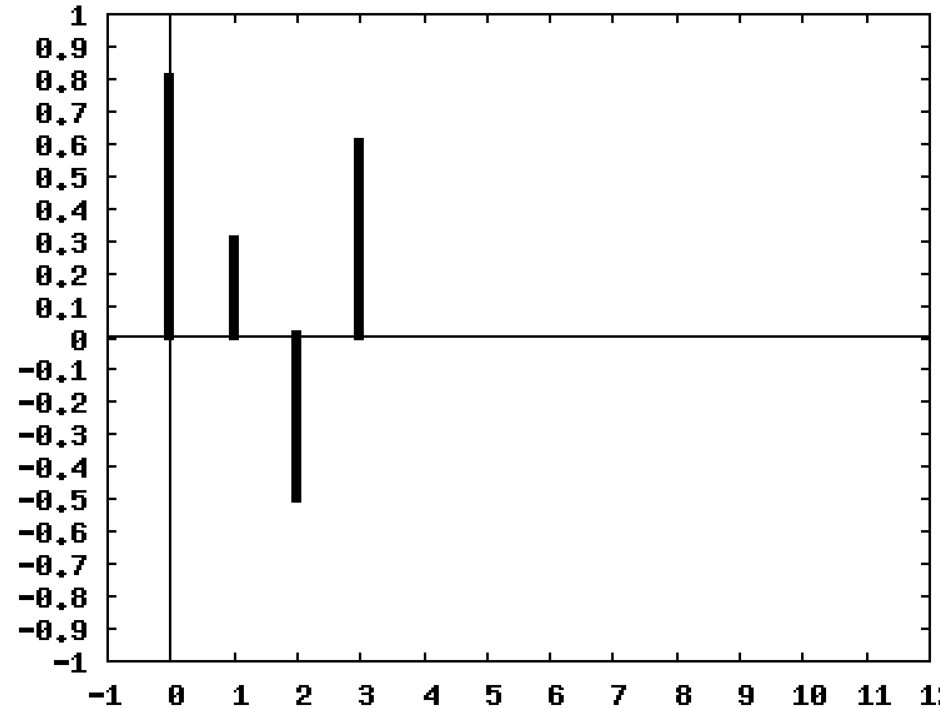

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

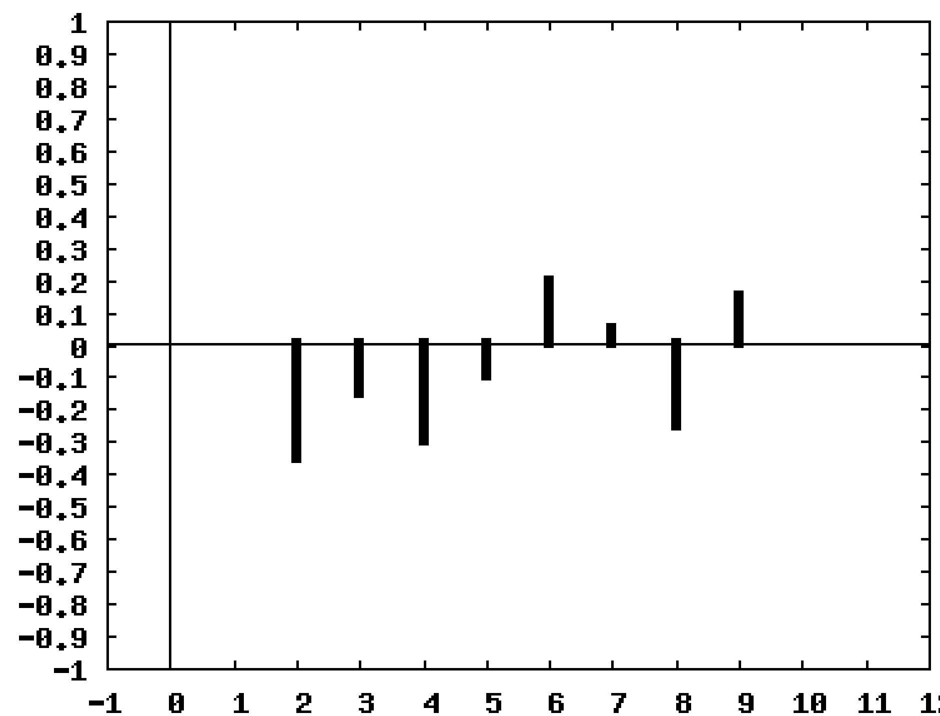

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

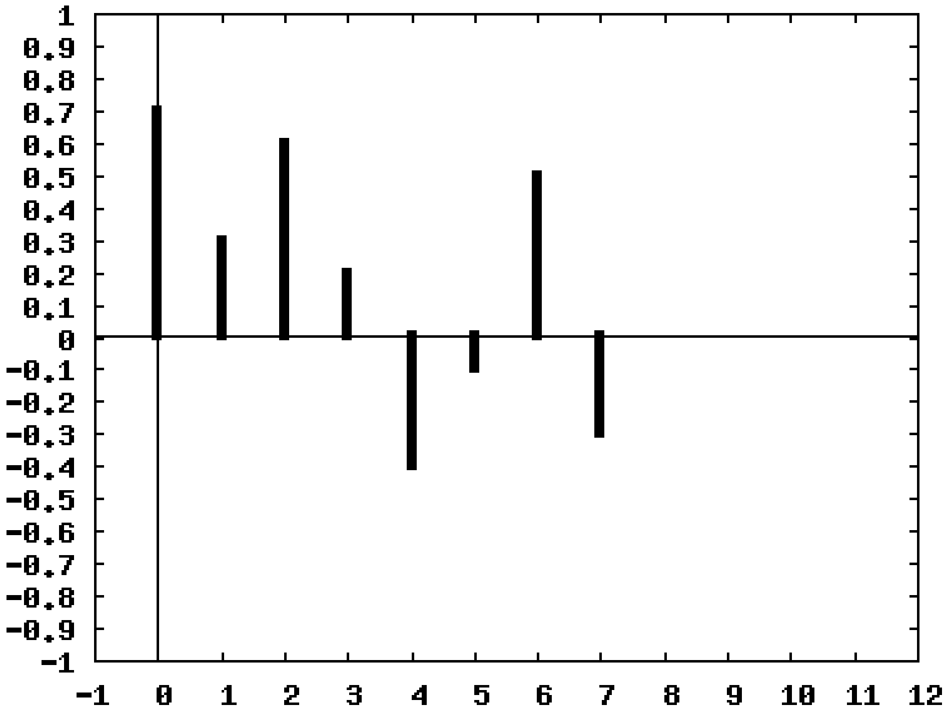

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

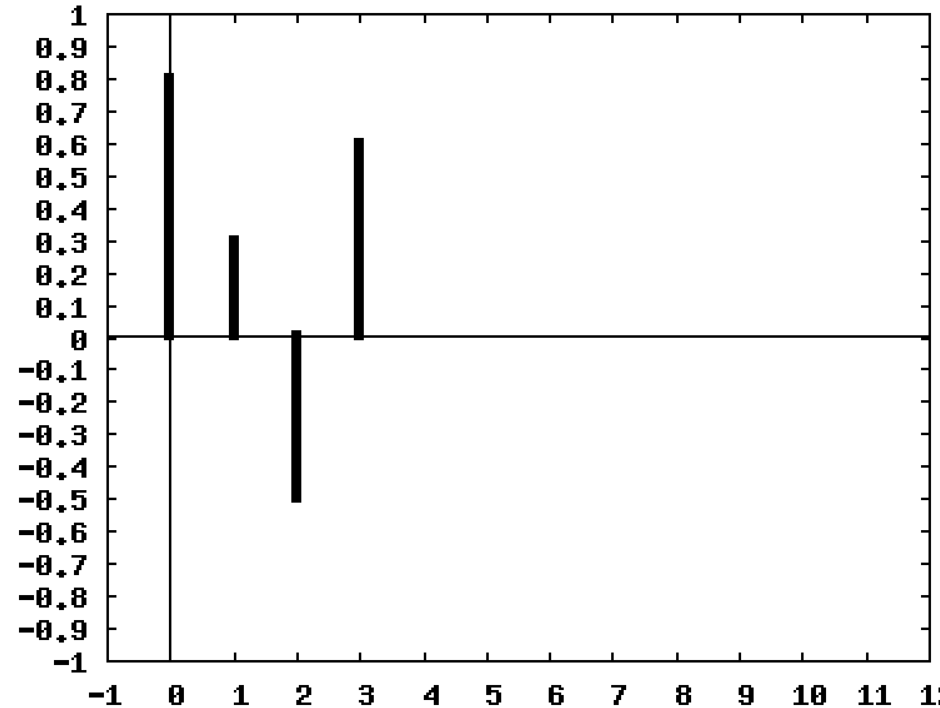

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

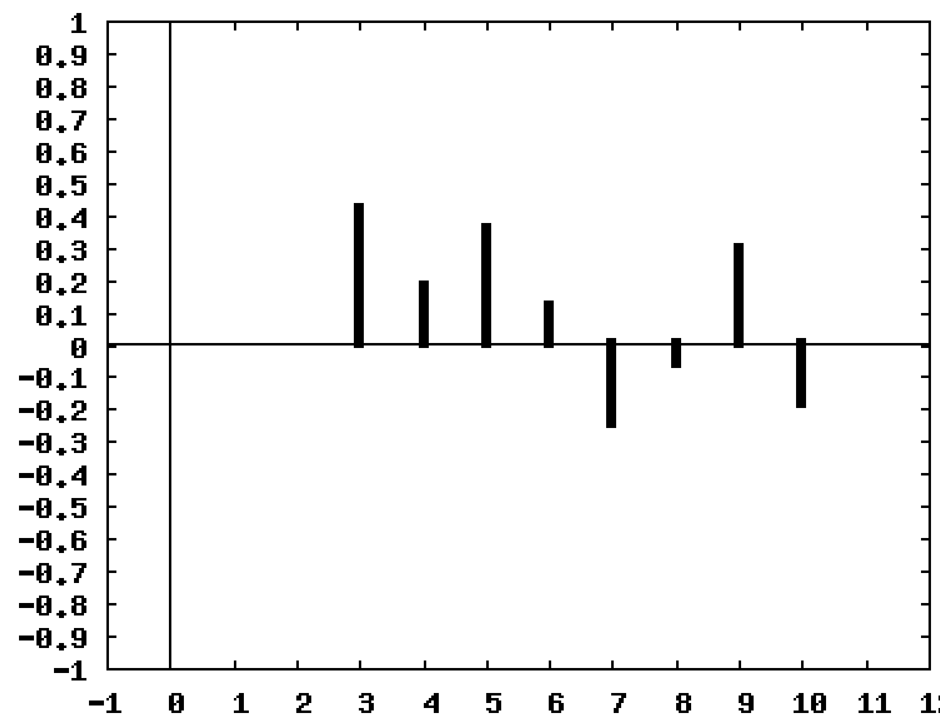

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

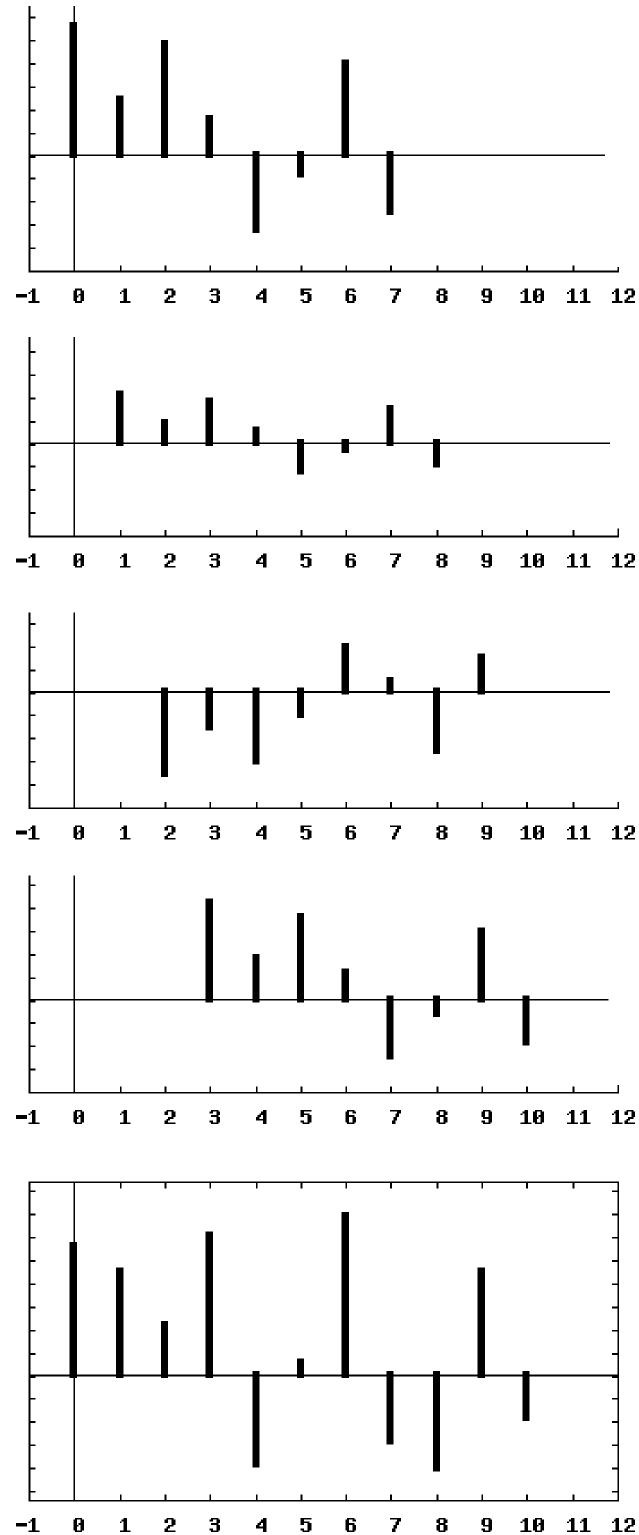

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12


1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

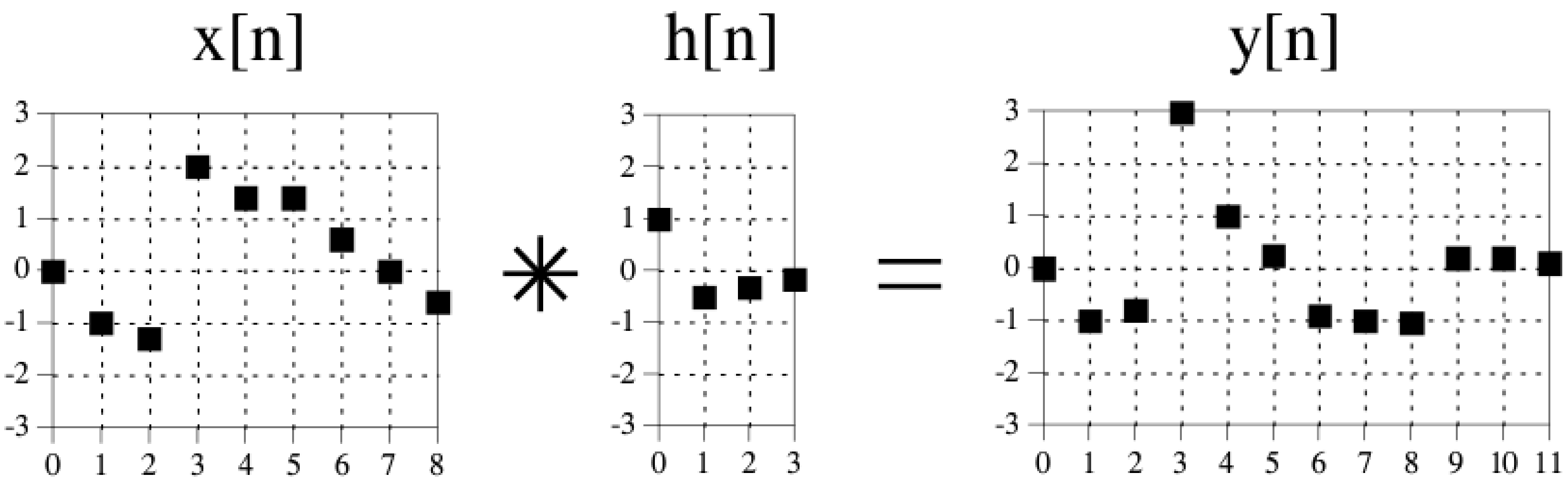

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12


1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

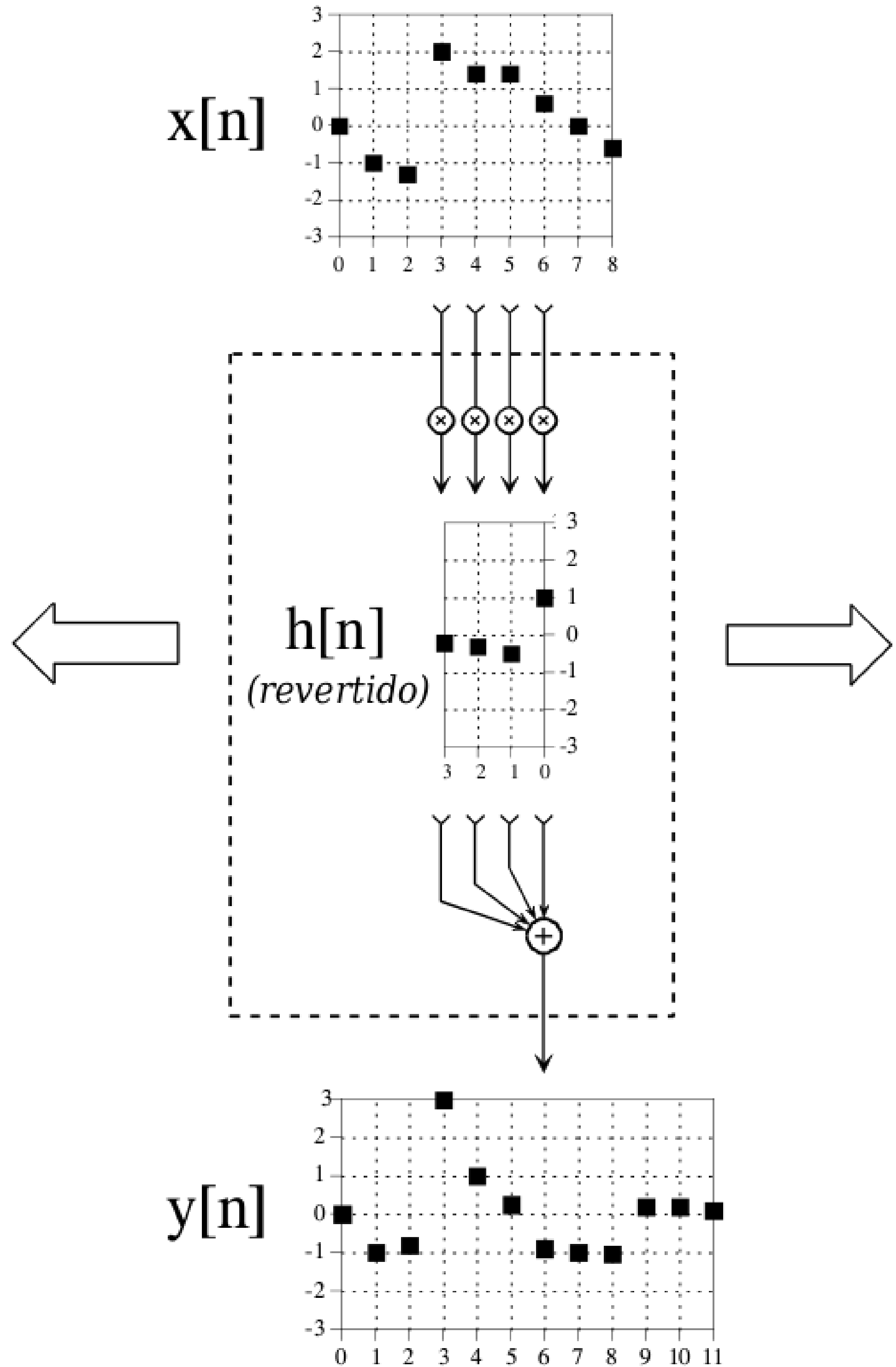

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

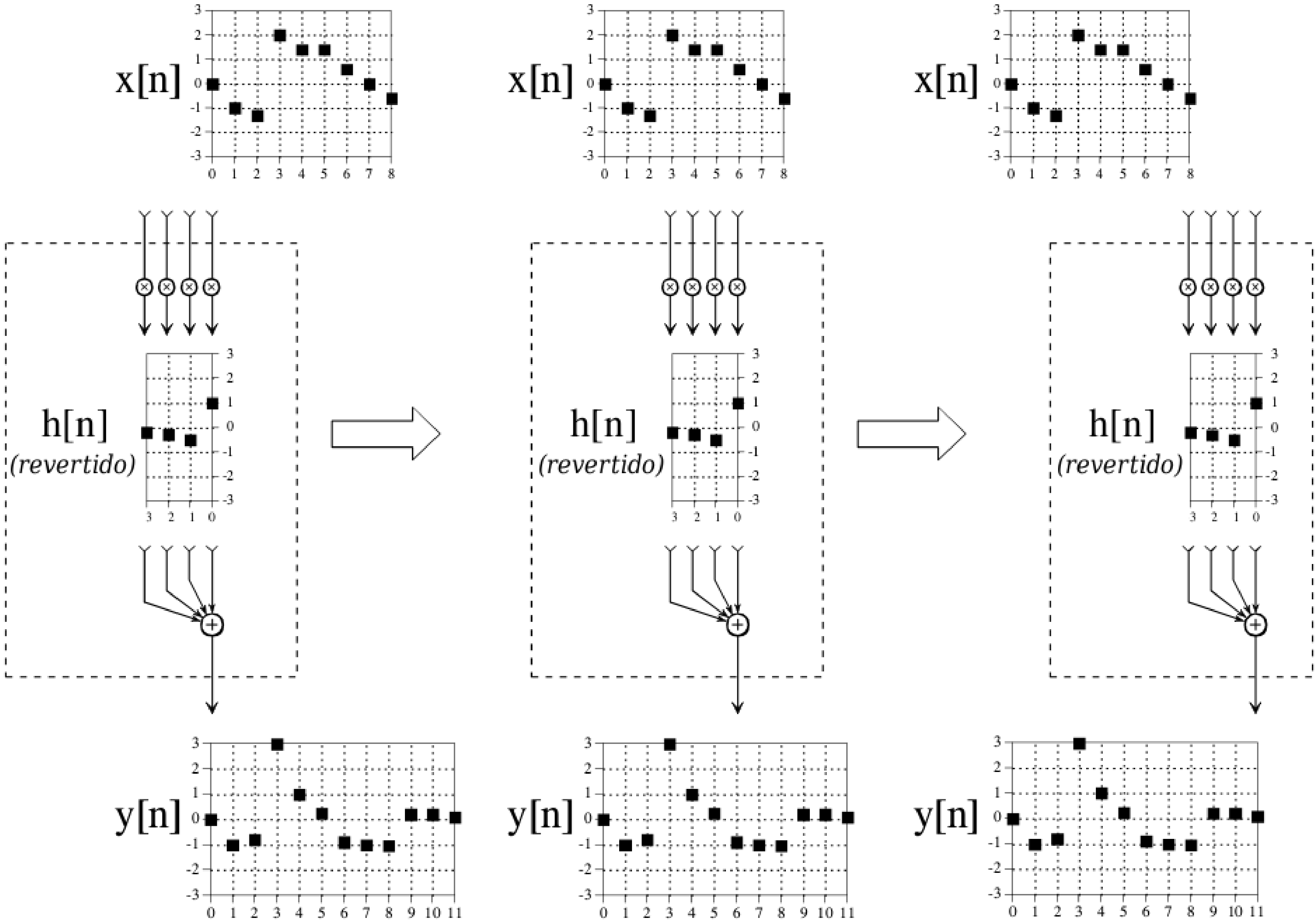

-1 0 1 2 3 4 5 6 7 8 9 10 11 12
-1 0 1 2 3 4 5 6 7 8 9 10 11 12
-1 0 1 2 3 4 5 6 7 8 9 10 11 12
-1 0 1 2 3 4 5 6 7 8 9 10 11 12
-1 0 1 2 3 4 5 6 7 8 9 10 11 12

- x[n] = N muestras
- h[n] = M muestras
- y[n] = N+M-1 muestras



$$y[i] = \sum_{j=0}^{M-1} h[j]\, x[i-j]$$


x[n]
h[n]
(revertido)
y[n]

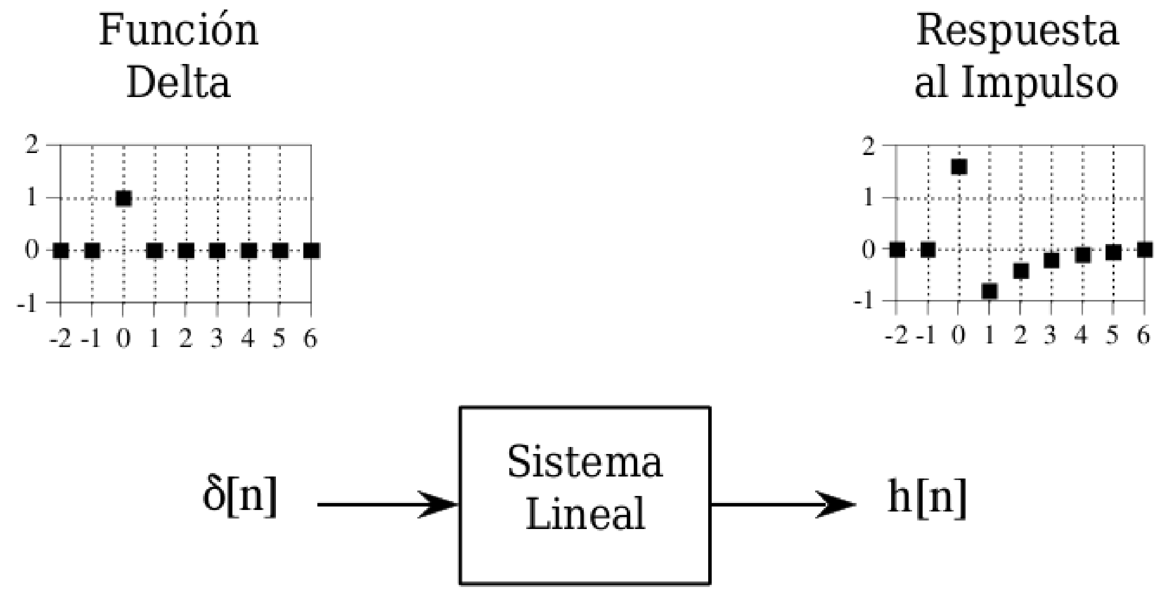


a. Cómputo de y[0]

b. Cómputo de y[6]

c. Cómputo de y[11]

propiedades de la convolución



$$x[n] * h[n] = y[n]$$

Convolucionar una señal $x_{[n]}$ con la respuesta impulsiva $h_{[n]}$ de un sistema dado equivale a pasar la señal $x_{[n]}$ por ese sistema.

- Multiplicar dos señales en el dominio del tiempo equivale a convolucionar sus espectros en el dominio de la frecuencia

$$x_1[n] \cdot x_2[n] = y[n]$$

$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

$$X_1[n] * X_2[n] = Y[n]$$

- Convolucionar dos señales en el dominio del tiempo equivale a multiplicar sus espectros en el dominio de la frecuencia

$$x_1[n] * x_2[n] = y[n]$$

$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

$$X_1[n] \cdot X_2[n] = Y[n]$$

Convolución rápida

- se pasan las dos señales al dominio de la frecuencia mediante la Transformada Rápida de Fourier (FFT)
- se multiplican los espectros
- se vuelve al dominio del tiempo mediante la transformada inversa de Fourier

Convolución rápida

- es un método más rápido y eficiente computacionalmente
- su ventaja sobre la convolución directa aumenta a medida de que crece el tamaño de las señales a convolucionar

propiedades de la convolución

- identidad

$$x[n] * \delta[n] = x[n]$$

propiedades de la convolución

- identidad

$$x[n] * \delta[n] = x[n]$$

- escalamiento (amplificación o atenuación)

$$x[n] * k\delta[n] = kx[n]$$

propiedades de la convolución

- identidad

$$x[n] * \delta[n] = x[n]$$

- escalamiento (amplificación o atenuación)

$$x[n] * k\delta[n] = kx[n]$$

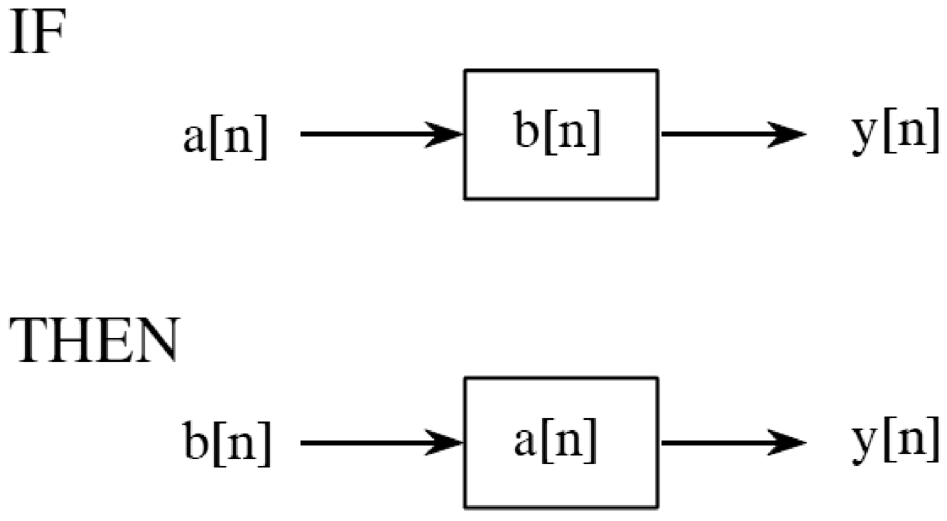
- desplazamiento (retardo o avance)

$$x[n] * \delta[n+s] = x[n+s]$$

propiedades matemáticas de la convolución

- conmutativa

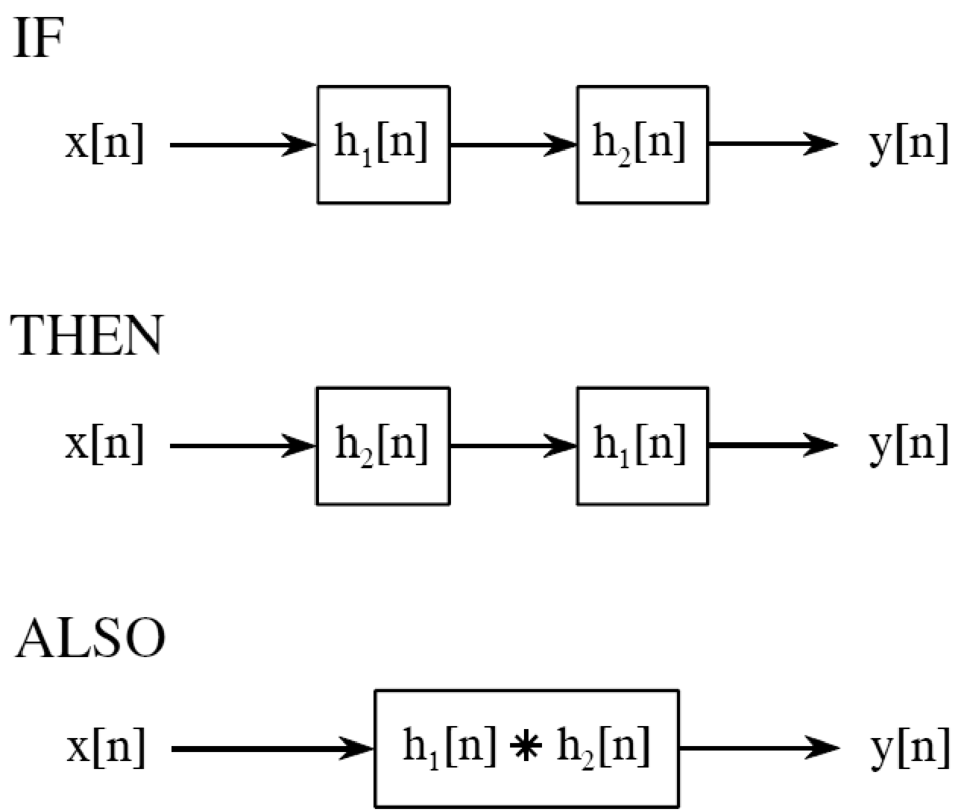
$$a[n] * b[n] = b[n] * a[n]$$



propiedades matemáticas de la convolución

- asociativa

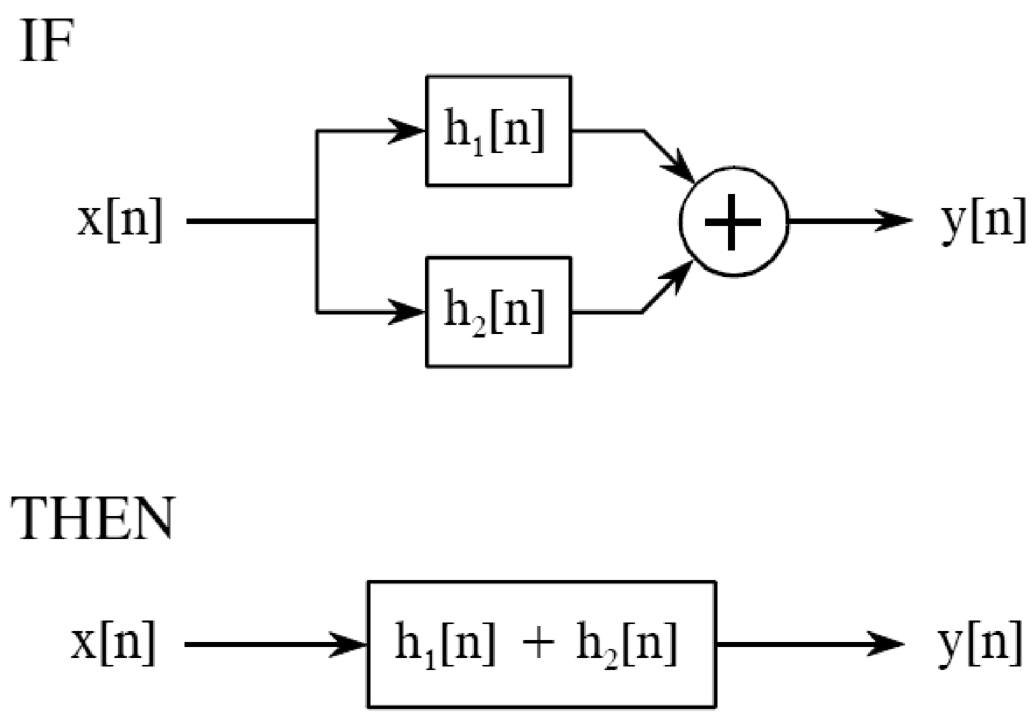
$$(a[n] * b[n]) * c[n] = a[n] * (b[n] * c[n])$$



propiedades matemáticas de la convolución

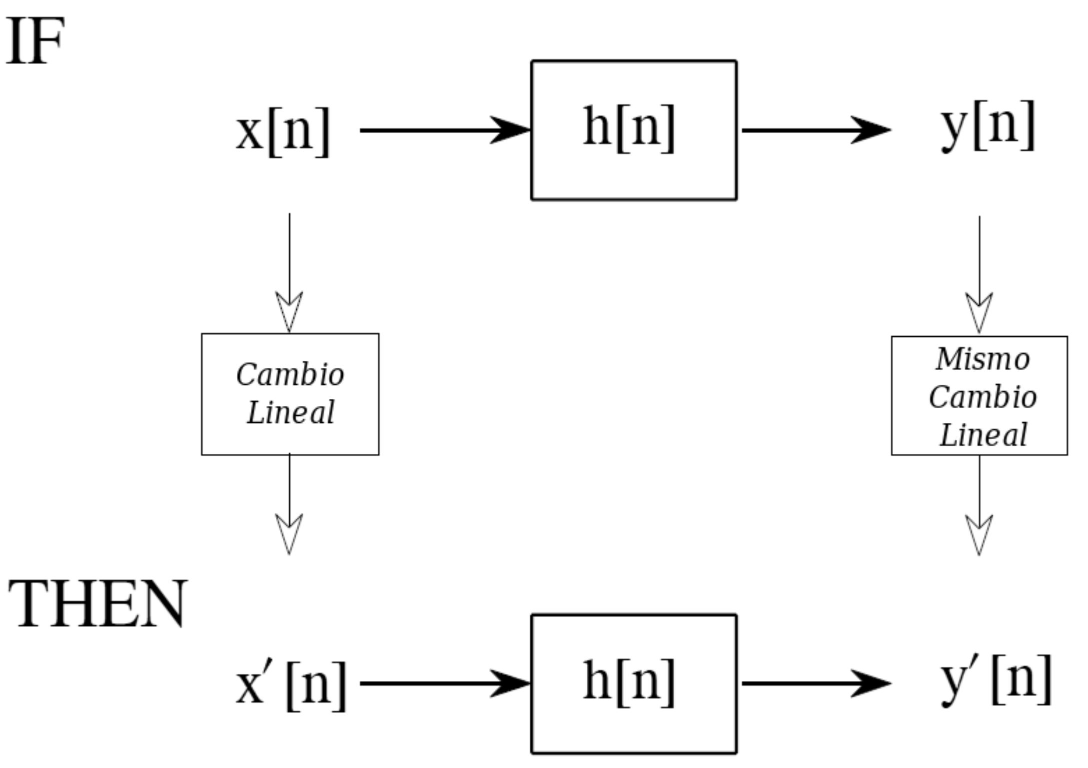
- distributiva

$$a[n] * b[n] + a[n] * c[n] = a[n] * (b[n] + c[n])$$

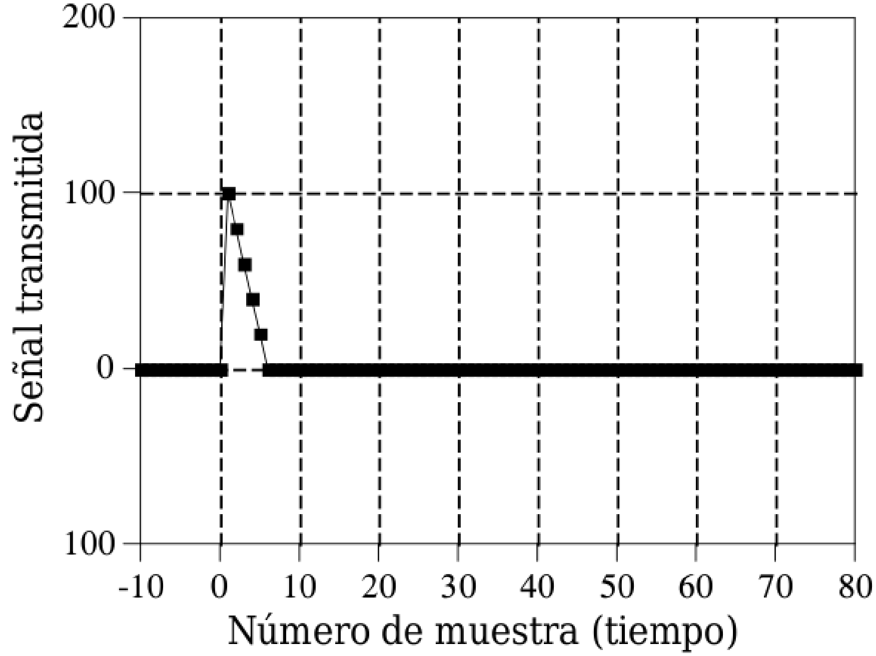


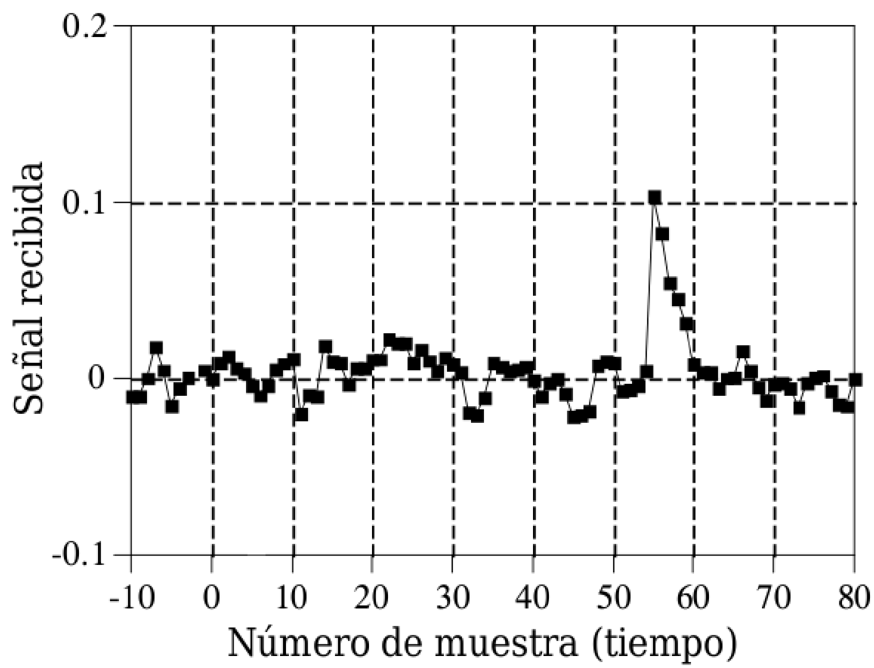
propiedades matemáticas de la convolución

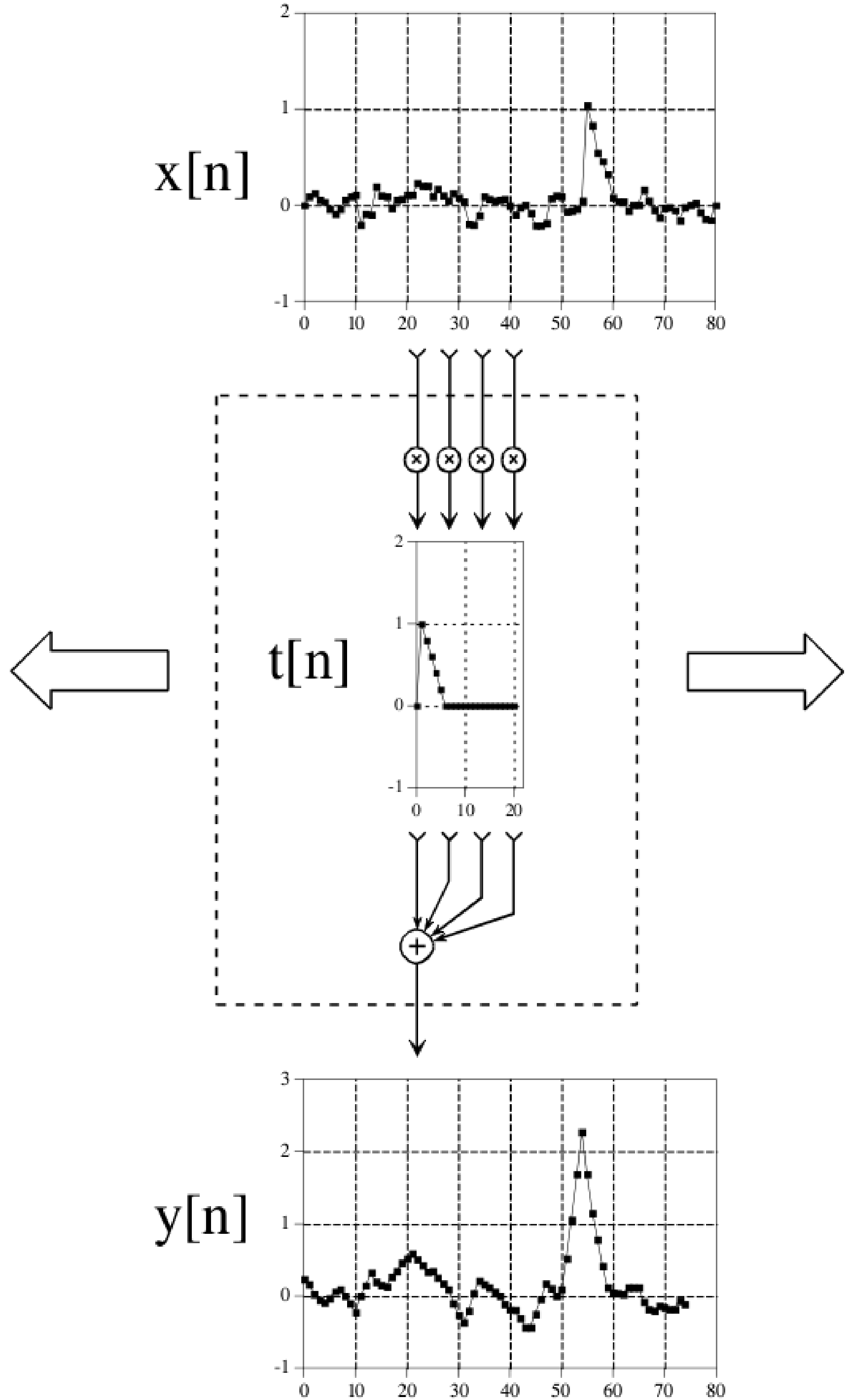
- transferencia entrada-salida



correlación


200
100
0
100
-10
0
10
20
30
40
50
60
70
80
Señal transmitida
Número de muestra (tiempo)


0.2
0.1
0
-0.1
-10
0
10
20
30
40
50
60
70
80
Señal recibida
Número de muestra (tiempo)


x[n]
t[n]
y[n]

$$y[i] = \sum_{j=0}^{M-1} h[j]\, x[i+j]$$