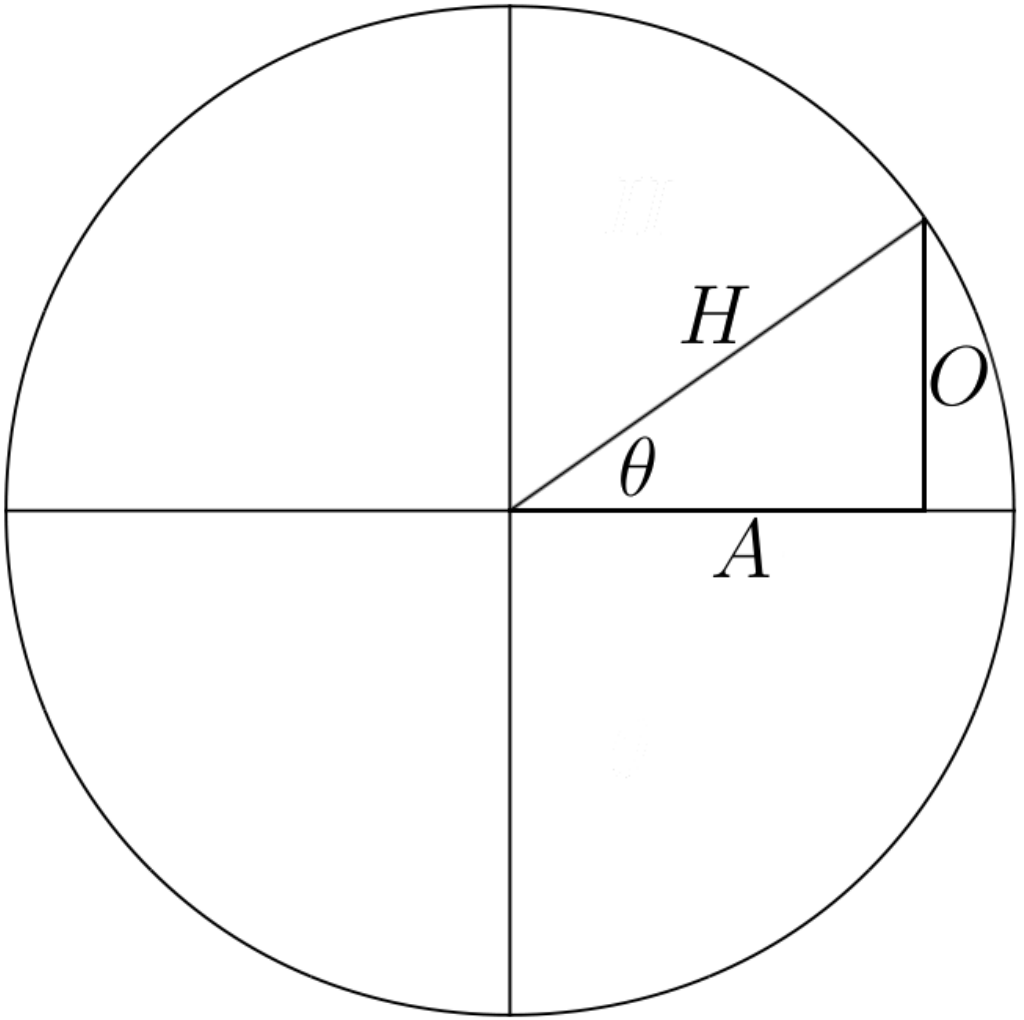
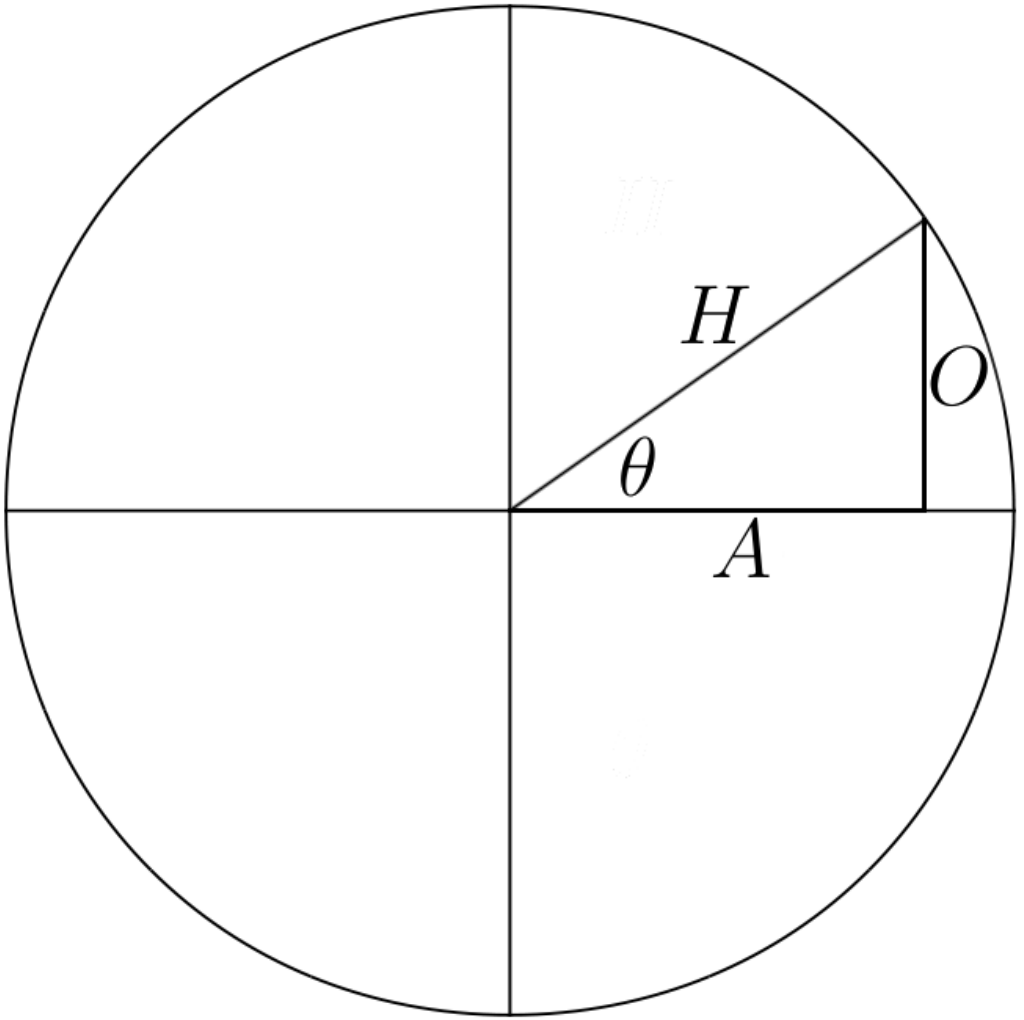
Introducción a la Teoría del Procesamiento Digital de Señales de Audio

clase 3

trigonometría

seno - coseno

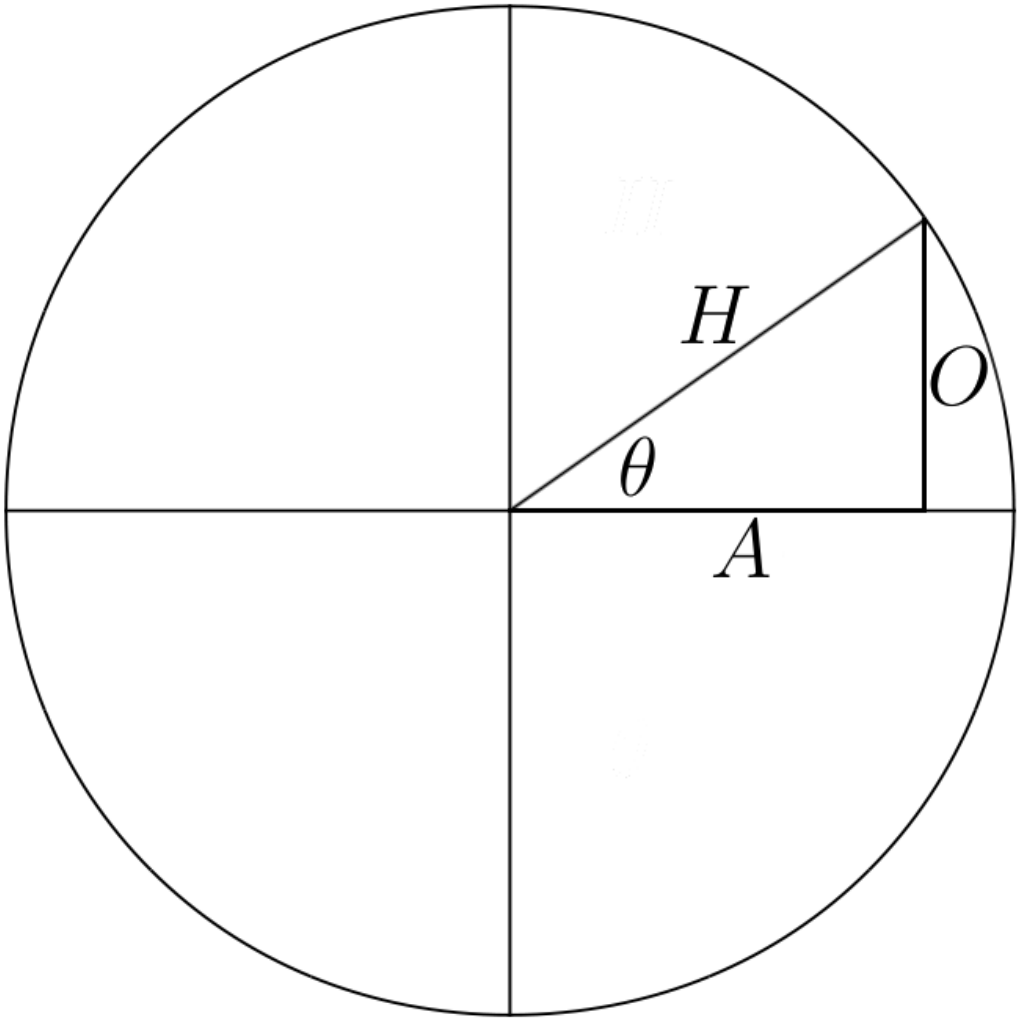




$$\sin(\theta) = \frac{O}{H}$$

$$\cos(\theta) = \frac{A}{H}$$

en el círculo unidad, H = 1

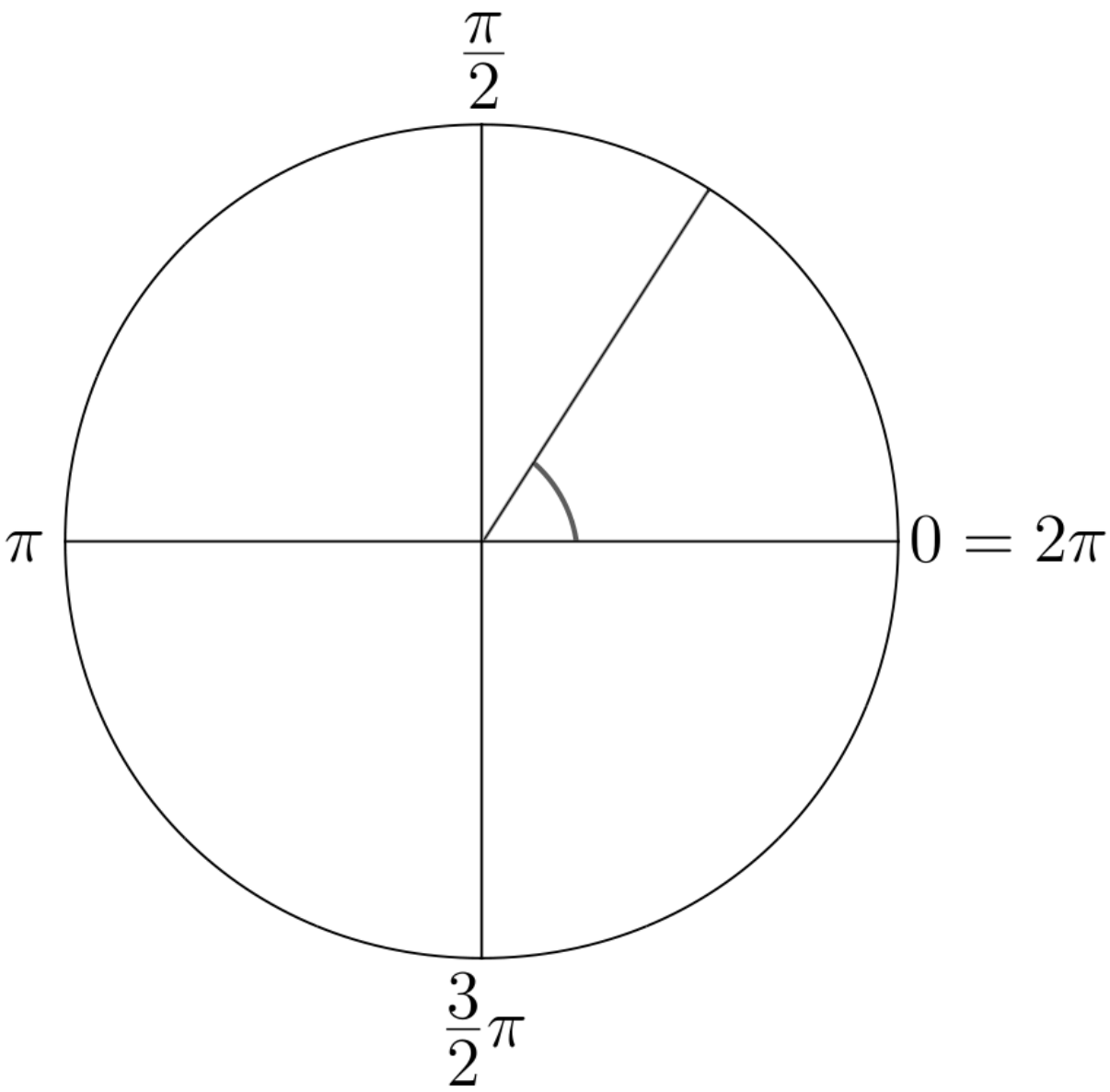


$$\sin(\theta) = O$$

$$\cos(\theta) = A$$

radián

ángulo cuyo arco mide un radio

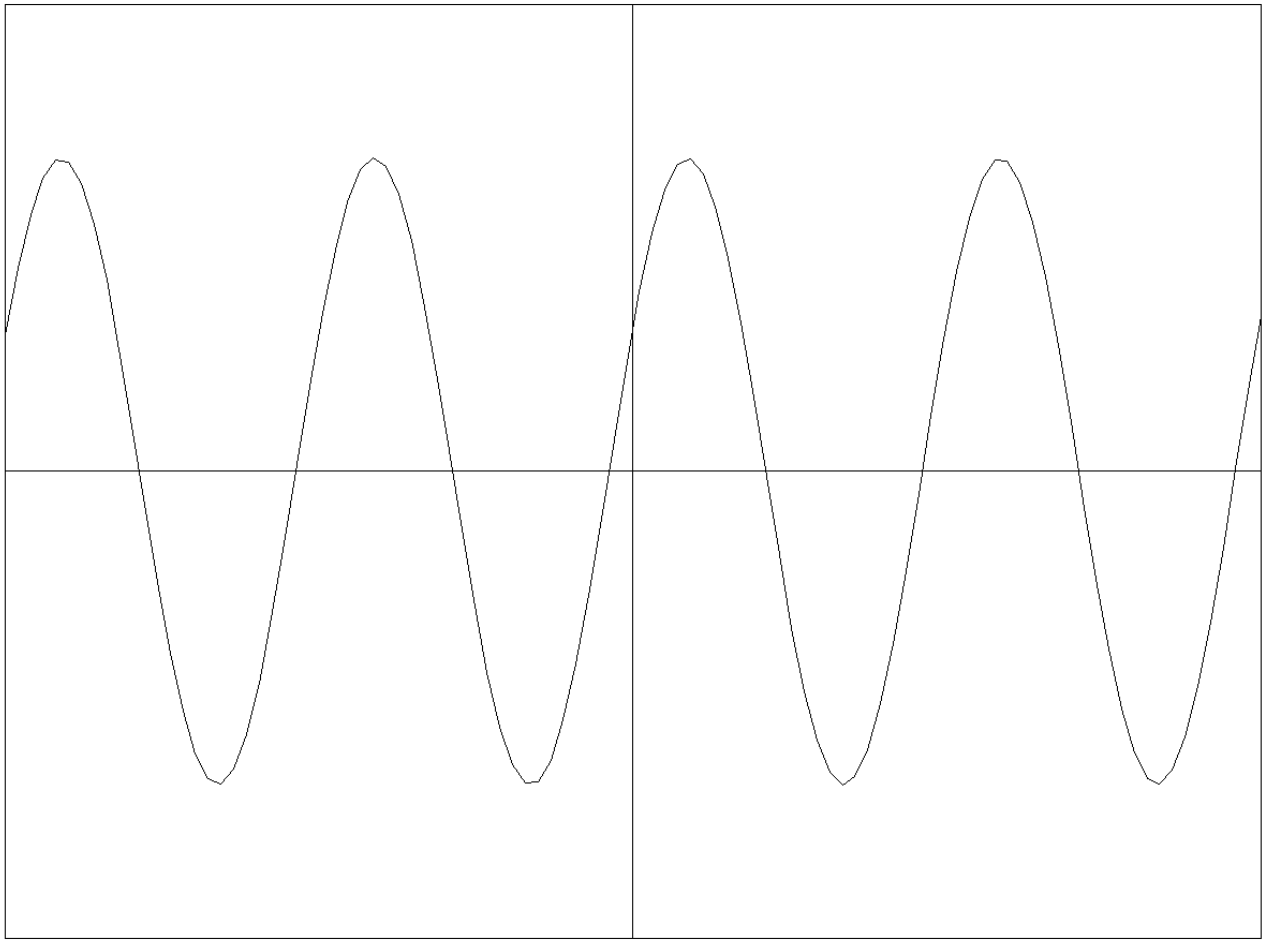


sinusoide

dominio continuo (real)

$$f(t) = A\sin(2\pi f t + \phi)$$

$$f(t) = A\sin(\omega t + \phi)$$



sinusoide

dominio discreto (entero)

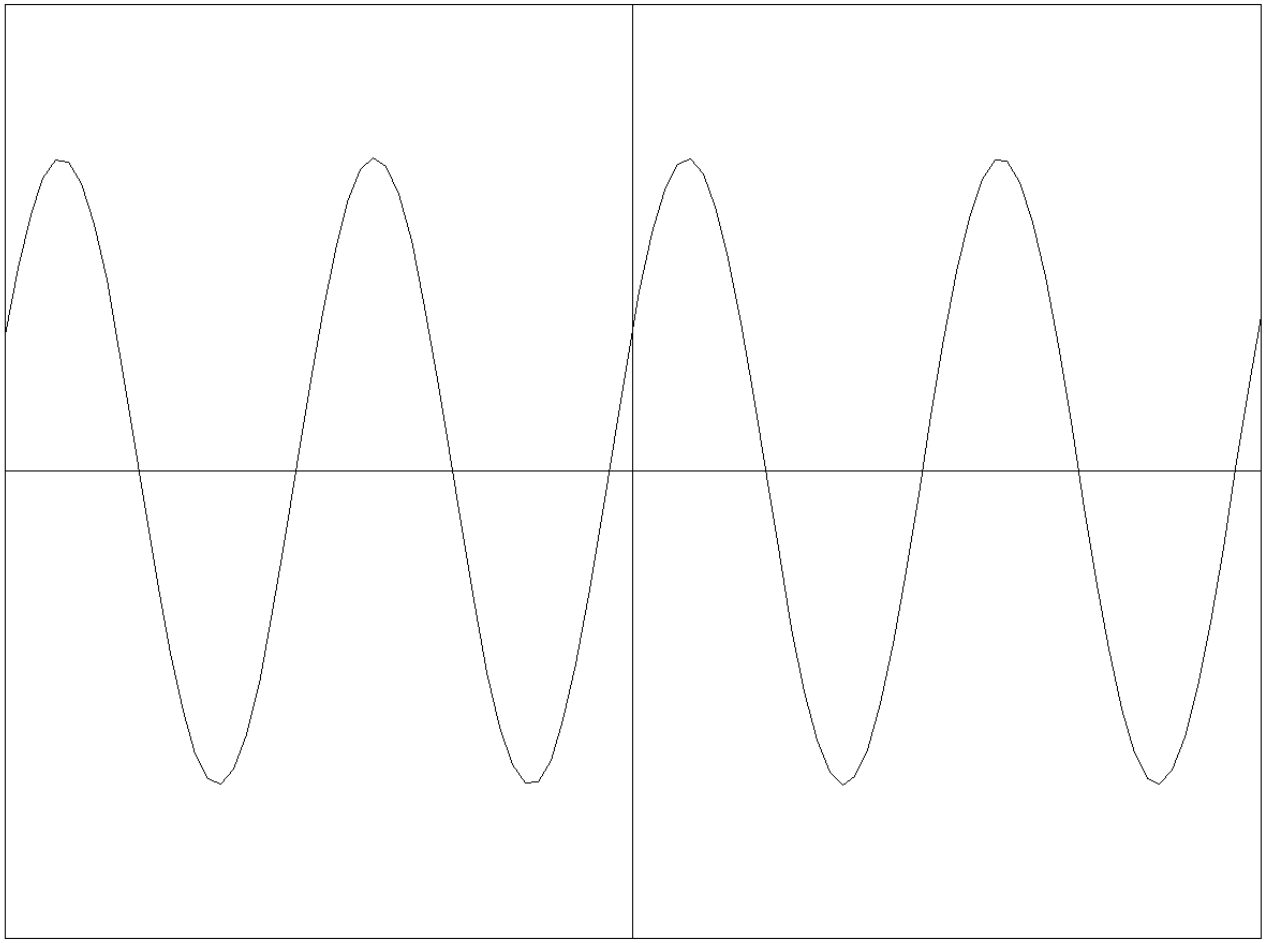
$$x[n] = A\sin(2\pi f n T + \phi) \qquad T = \frac{1}{F_m}$$

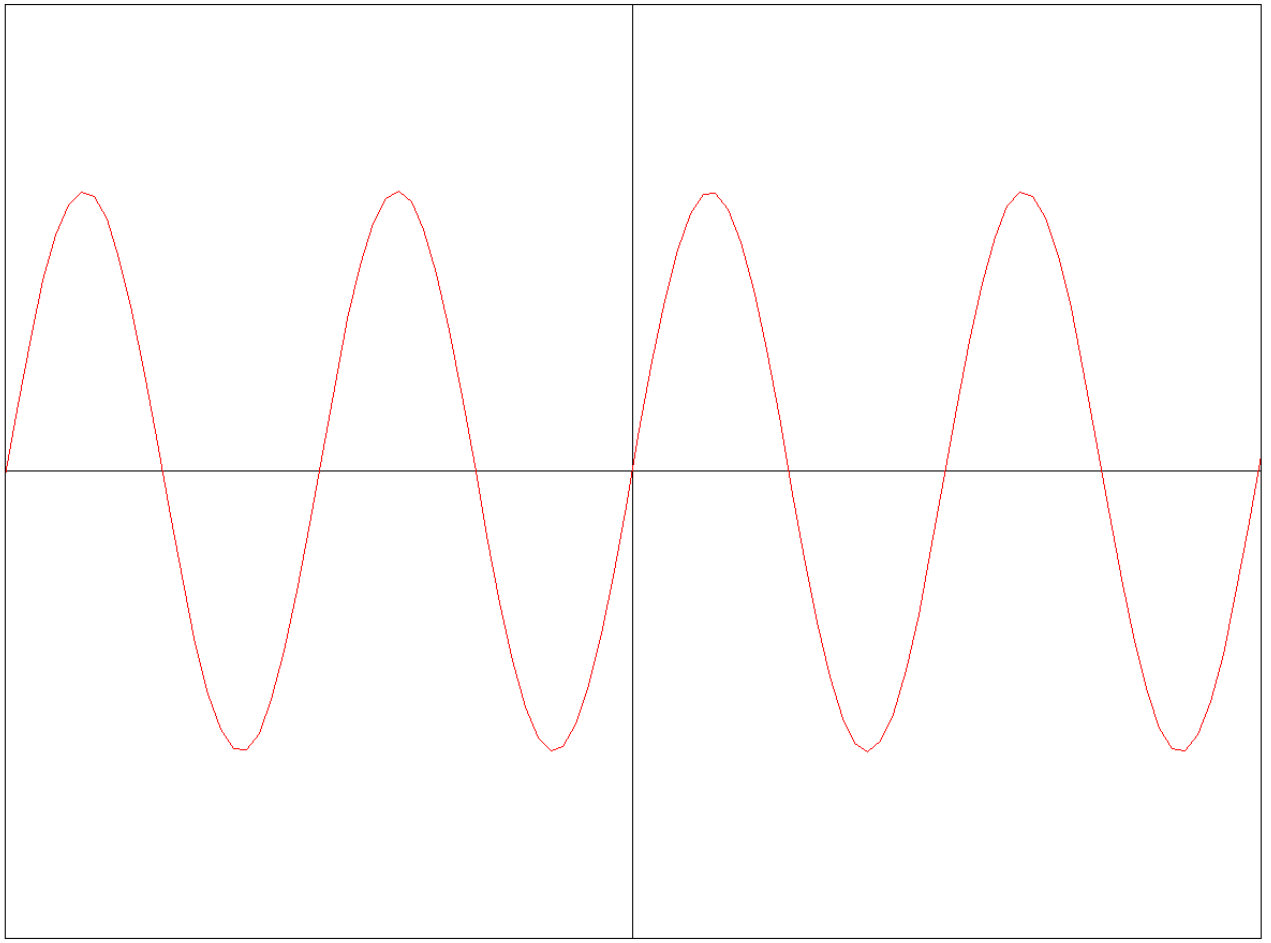
$$x[n] = A\sin(2\pi f n + \phi)$$

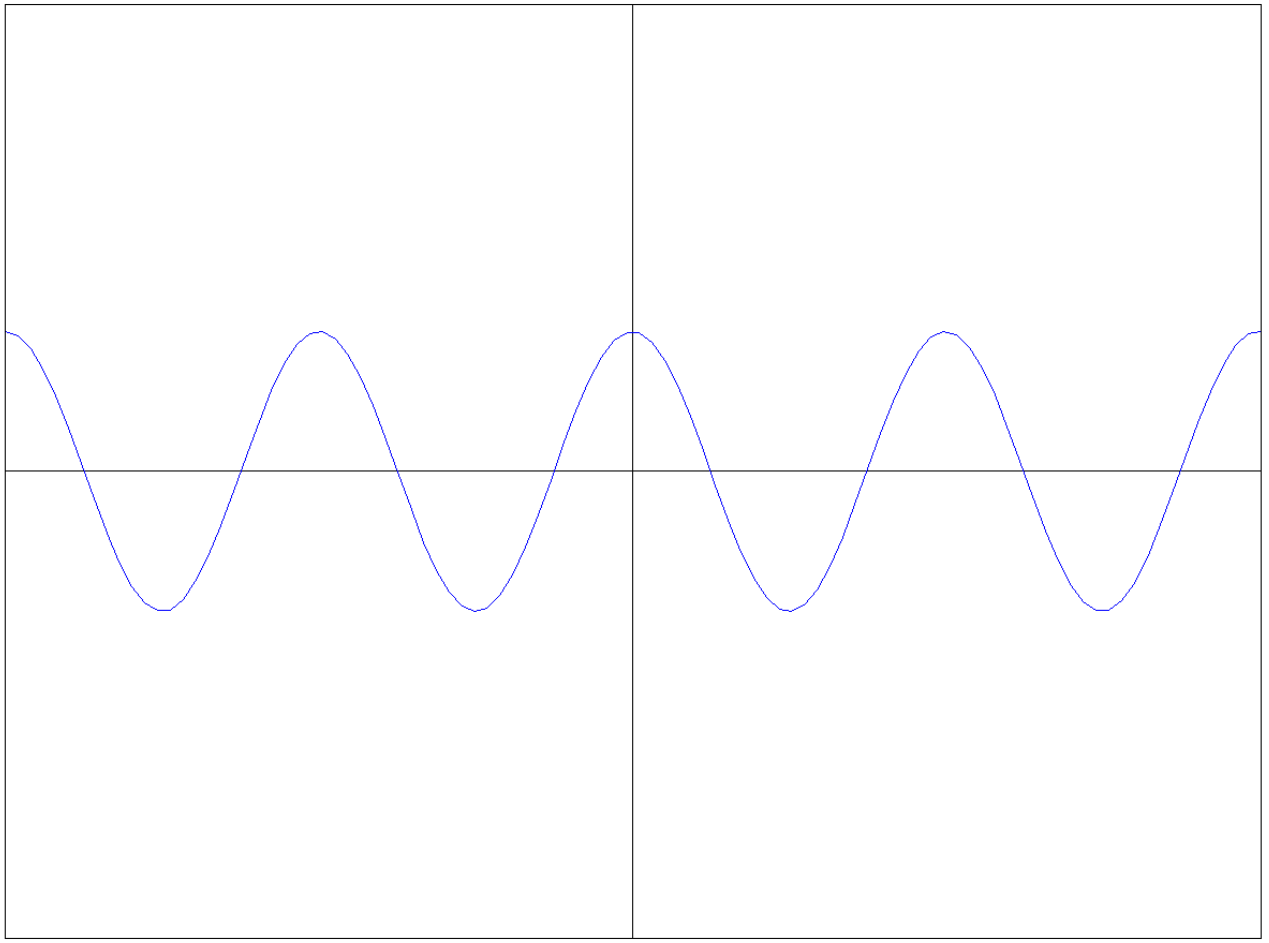
$$= A\sin(\omega n + \phi)$$

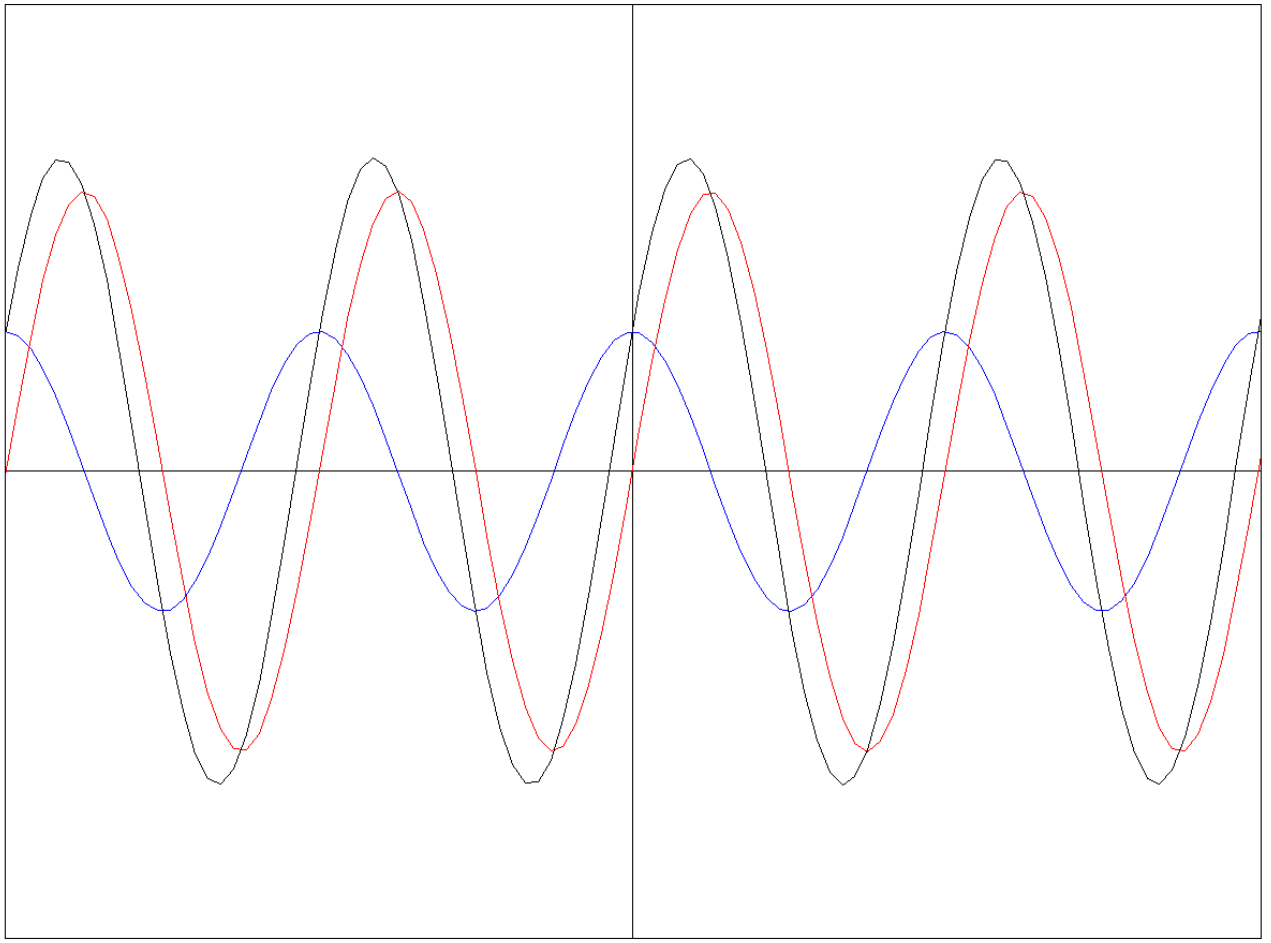
toda sinusoide se puede descomponer como la suma de un seno y un coseno

$$A\sin(\omega t + \phi) = a\cos\omega t + b\sin\omega t$$







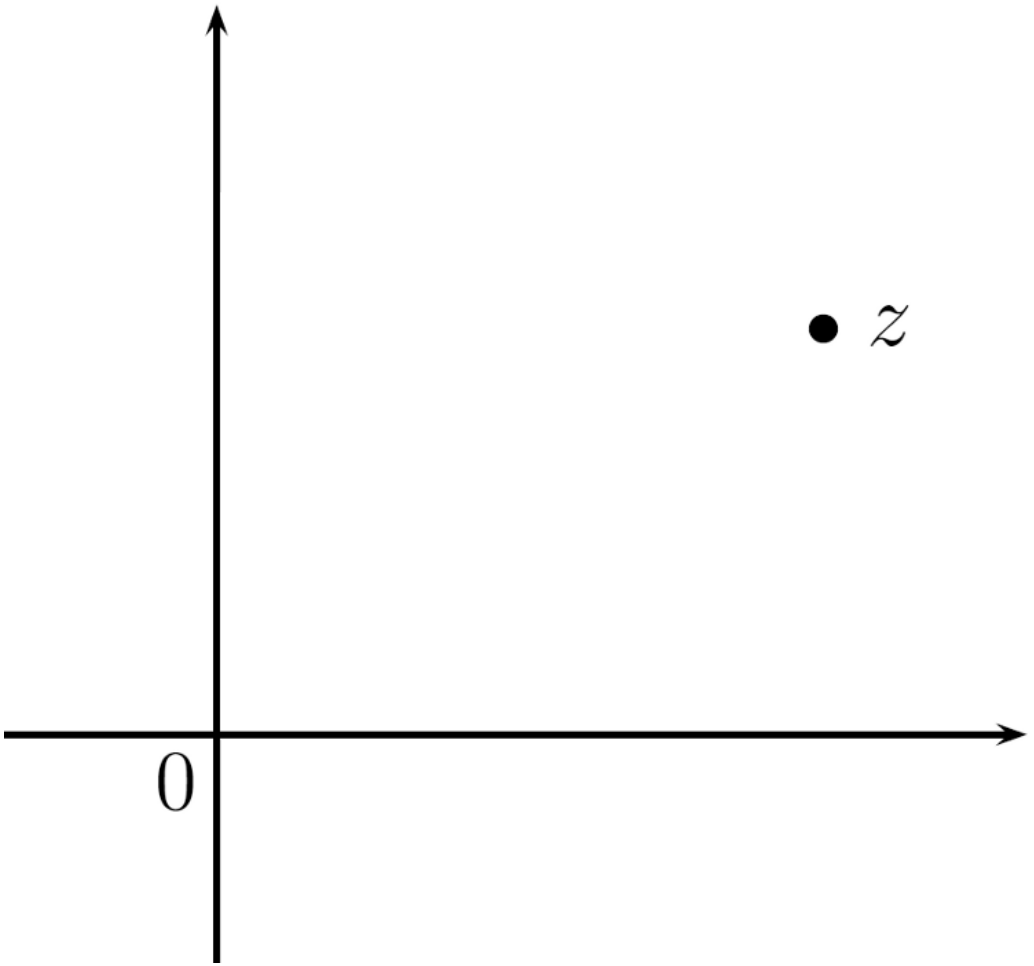


$$A\sin(\omega t + \phi) = a\cos\omega t + b\sin\omega t$$

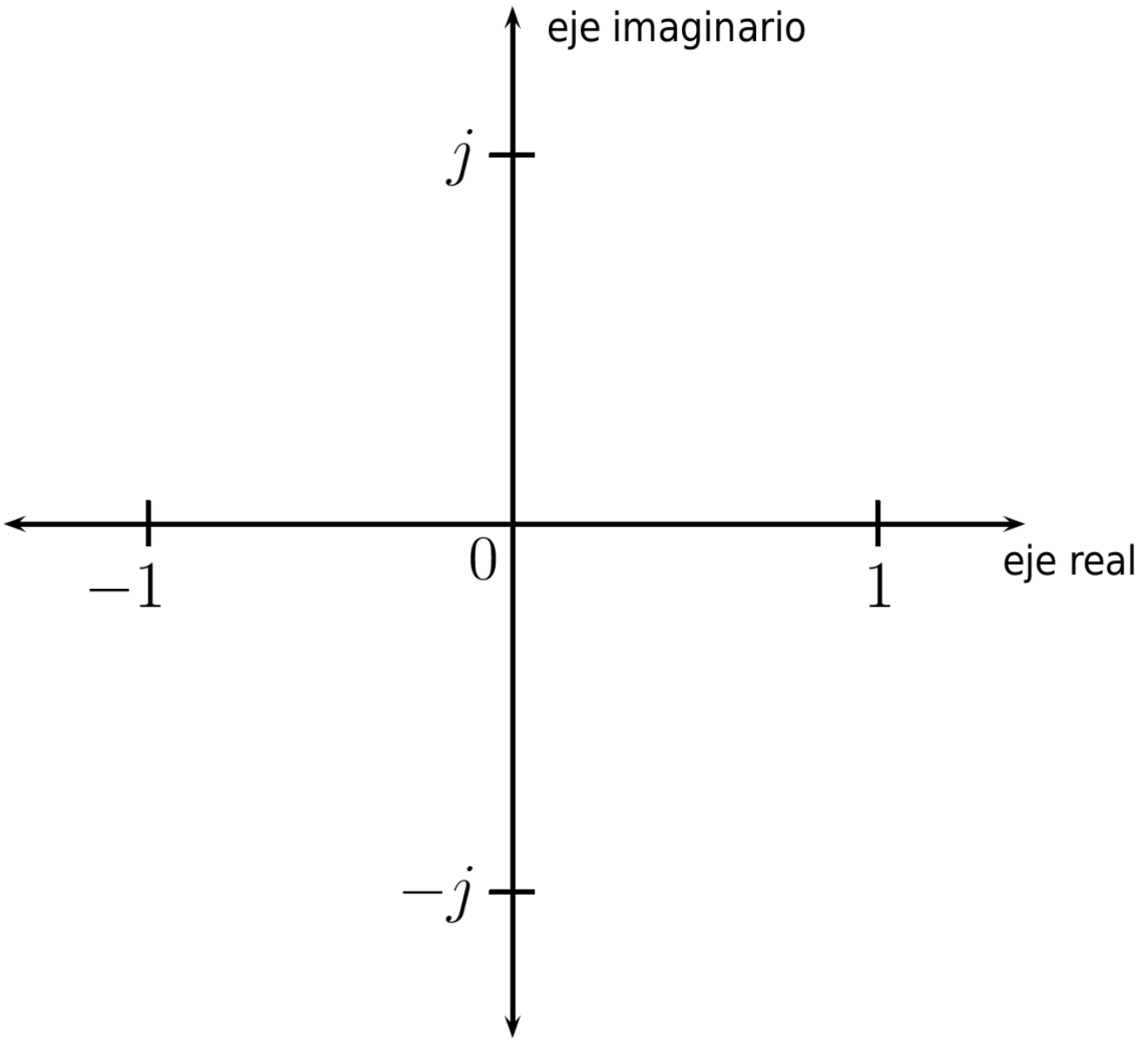
$$A = \sqrt{a^2 + b^2} \qquad \phi = \tan^{-1}\frac{b}{a}$$

$$a = A\cos\phi \qquad b = A\sin\phi$$

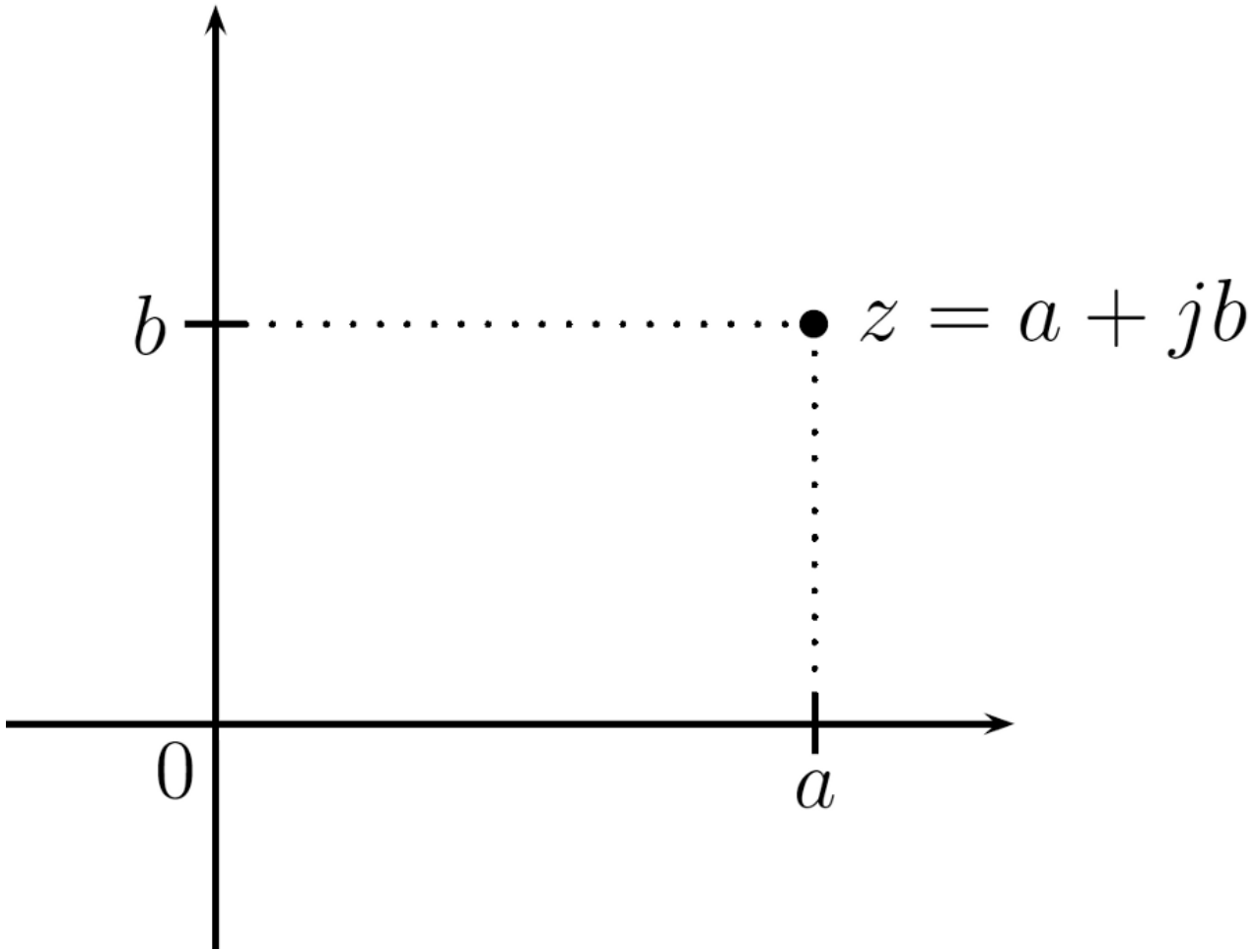
números complejos


z
0

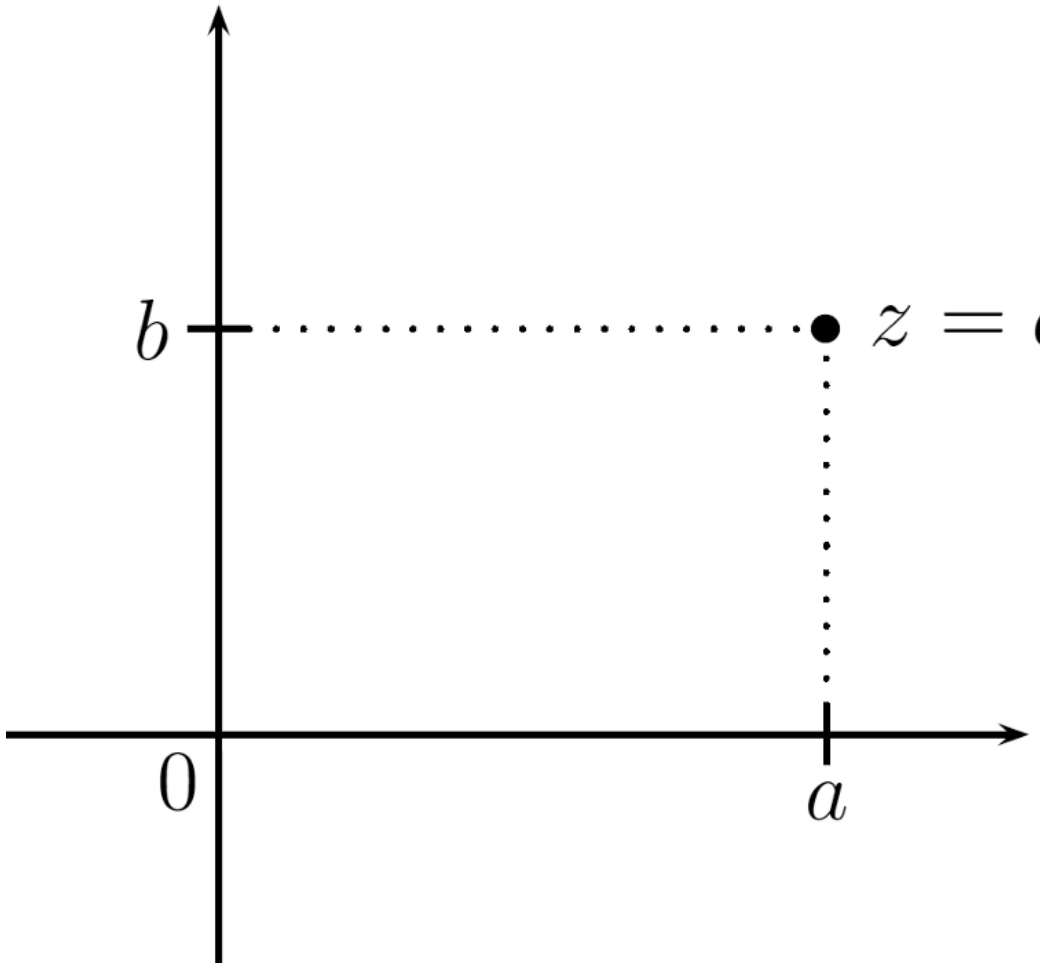
el plano complejo



notación algebraica (cartesiana)

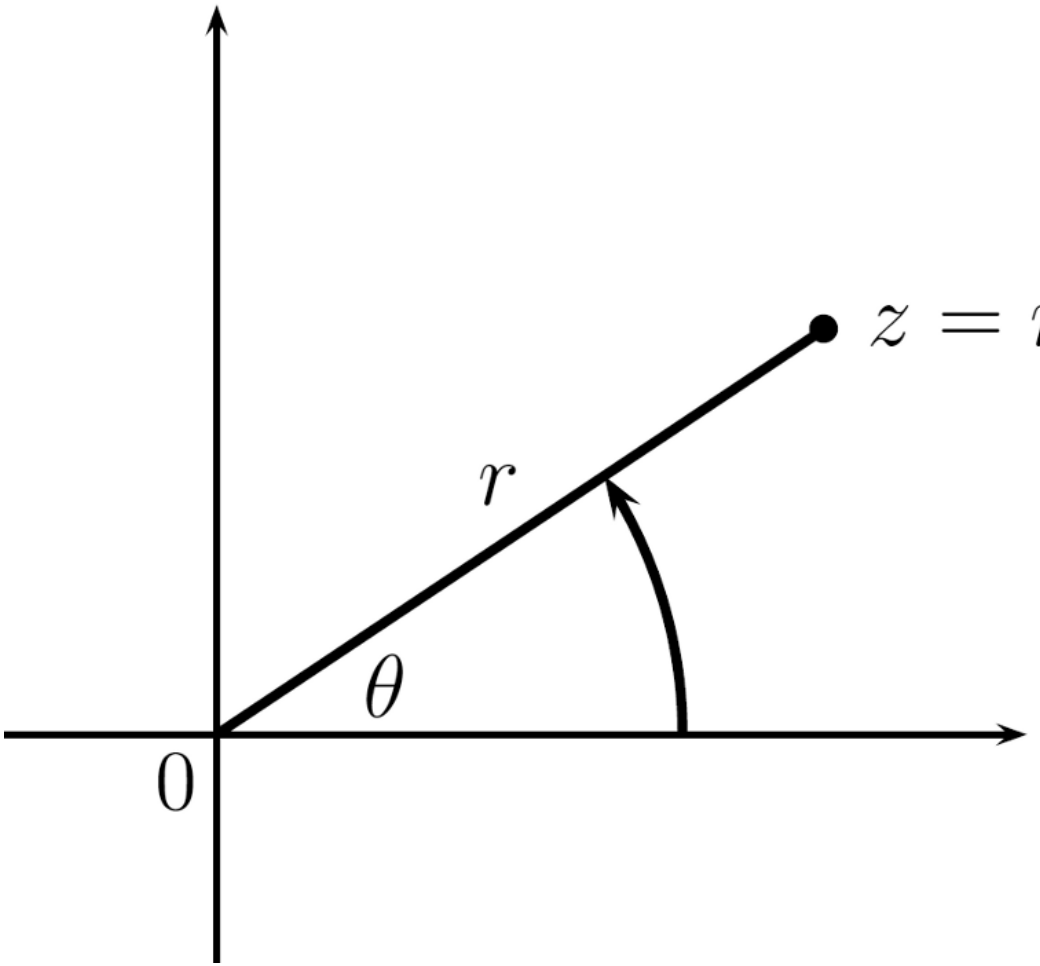


notación algebraica (cartesiana)

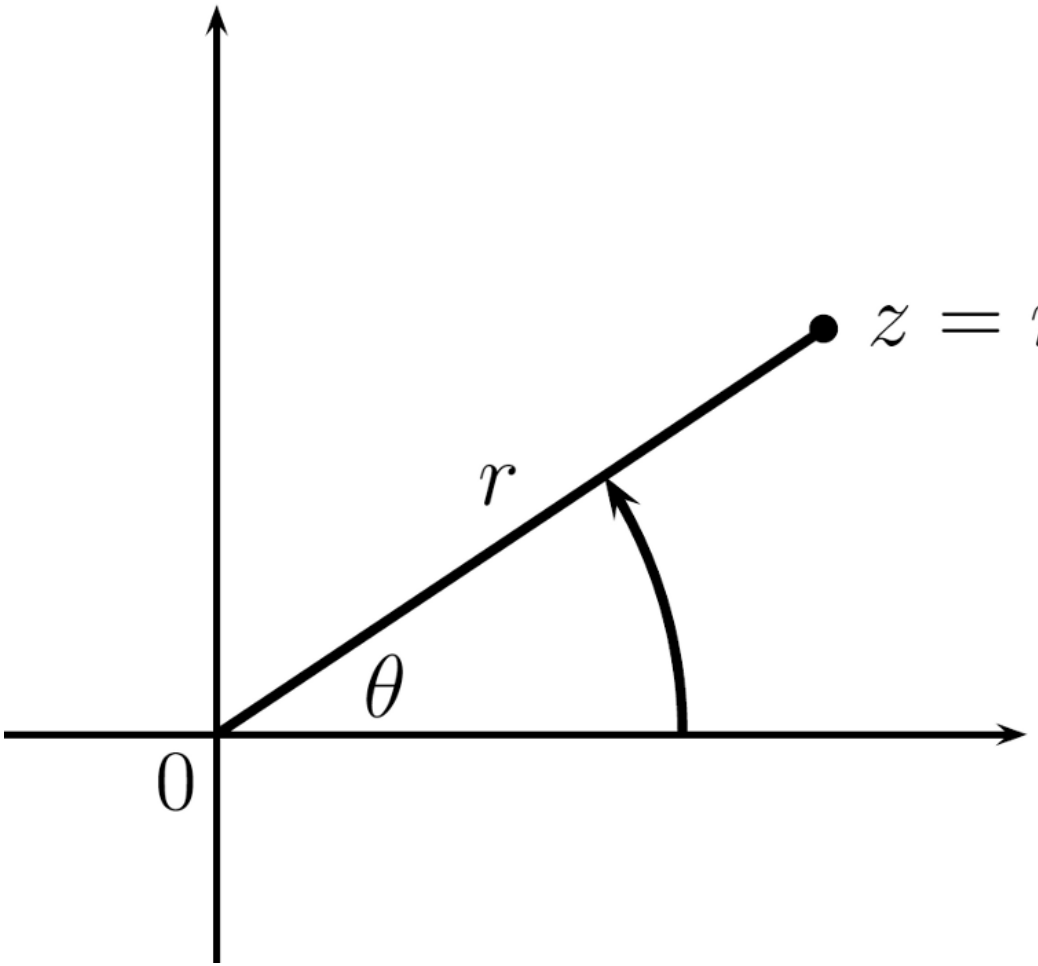


a = parte real
b = parte imaginaria

notación trigonométrica (polar)



notación trigonométrica (polar)



r = módulo (valor absoluto)
θ = argumento (radianes)

valor absoluto (módulo)

$$|z| = \sqrt{a^2 + b^2}$$

conversión de notaciones

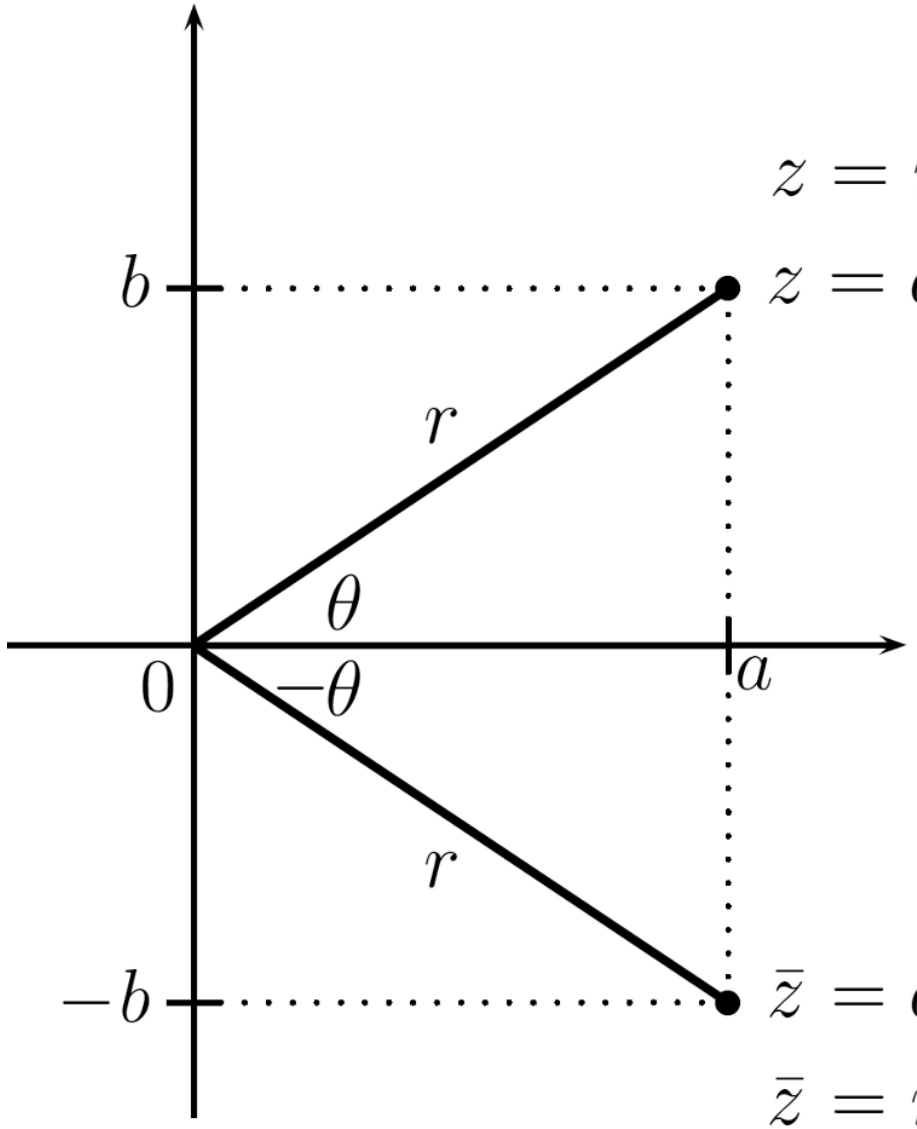
$$z = a + jb$$

$$z = r(\cos\theta + j\sin\theta)$$

$$a = r\cos\theta \quad b = r\sin\theta$$

$$r = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}\frac{b}{a}$$

conjugado de un complejo



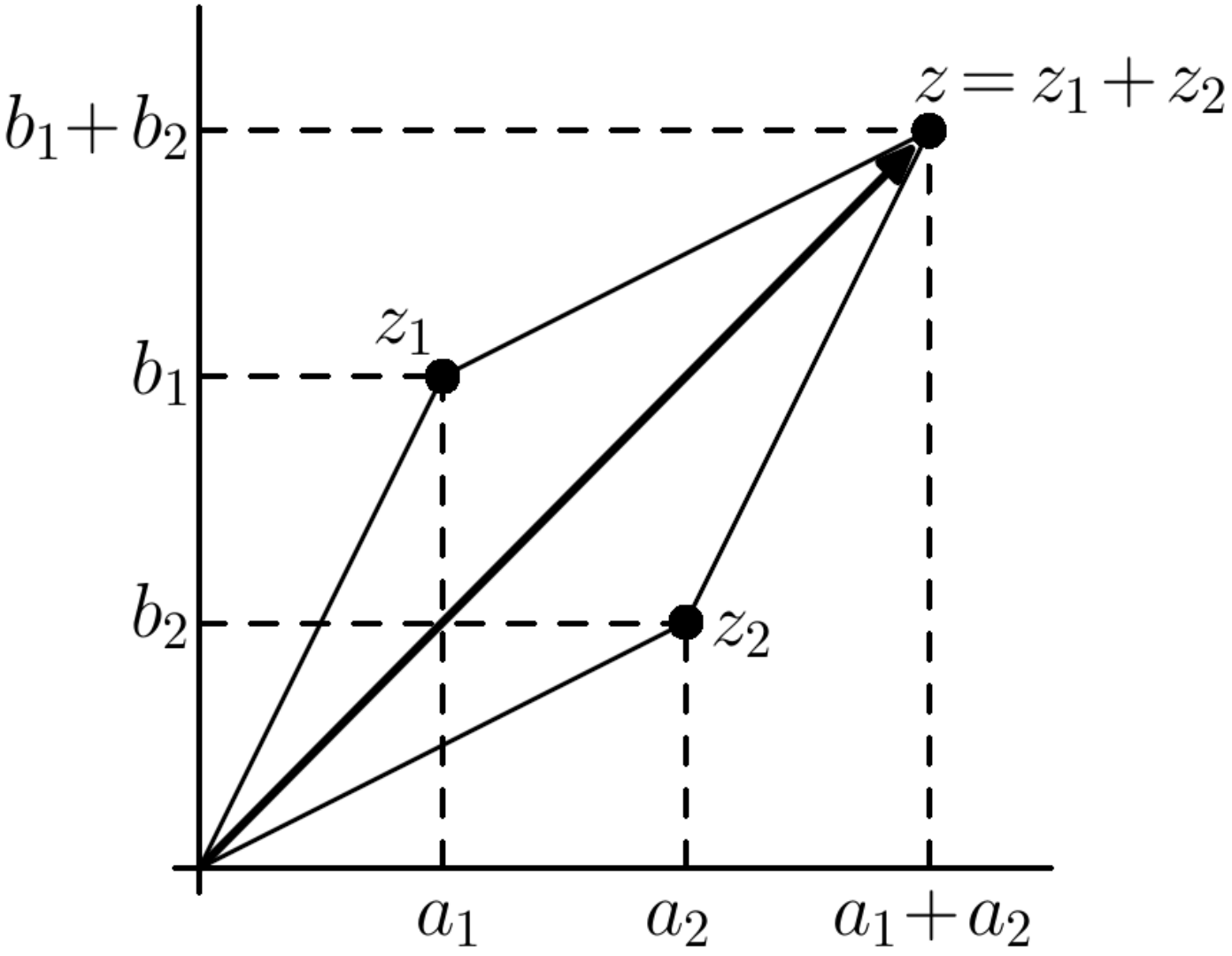
$$z = r(\cos\theta + j\sin\theta)$$

$$z = a + jb$$

$$\bar{z} = a - jb$$

$$\bar{z} = r(\cos\theta - j\sin\theta)$$

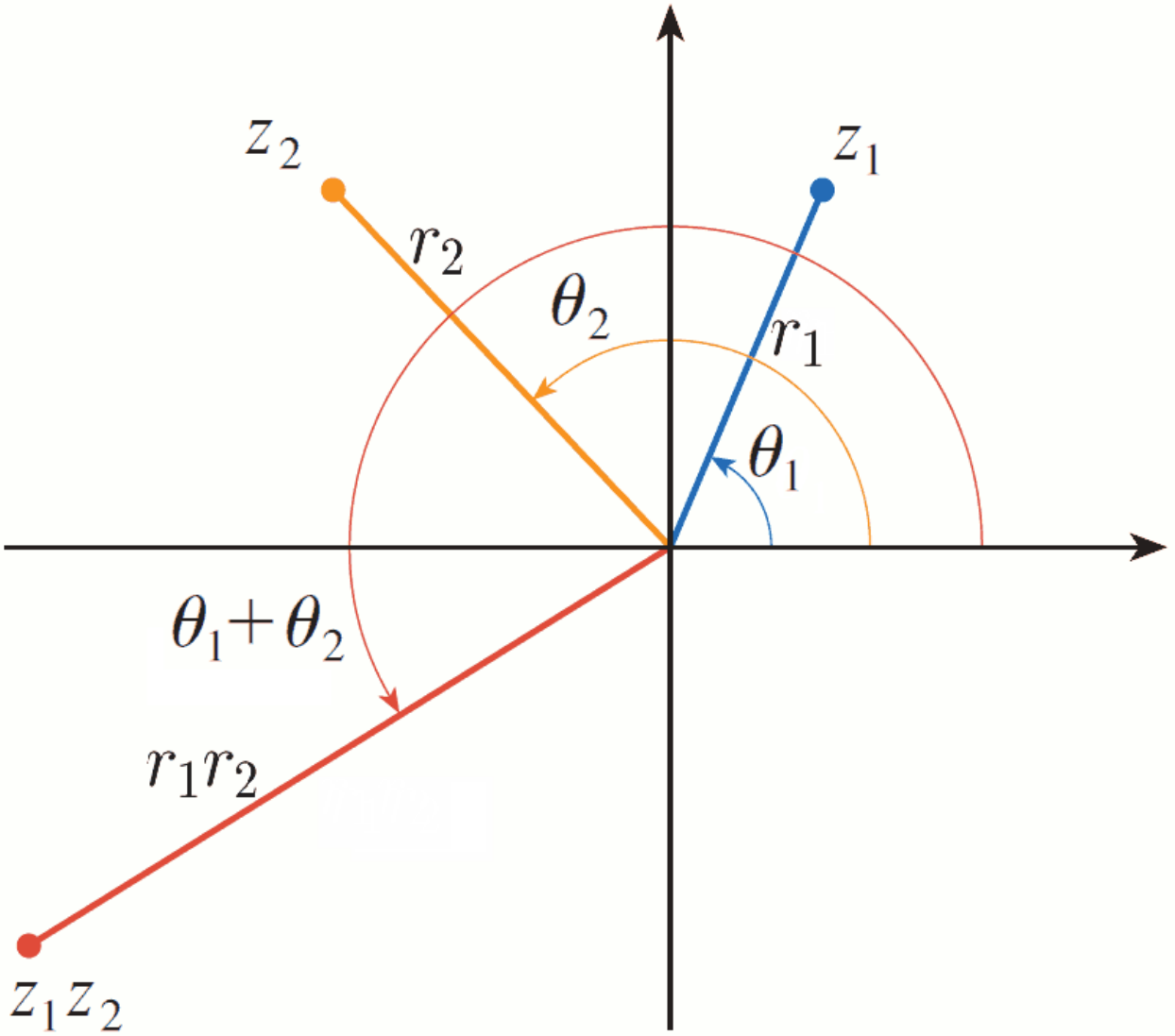
suma de complejos



suma de complejos

$$(a_1+jb_1)+(a_2+jb_2) = (a_1+a_2)+j(b_1+b_2)$$

multiplicación de complejos



multiplicación de complejos

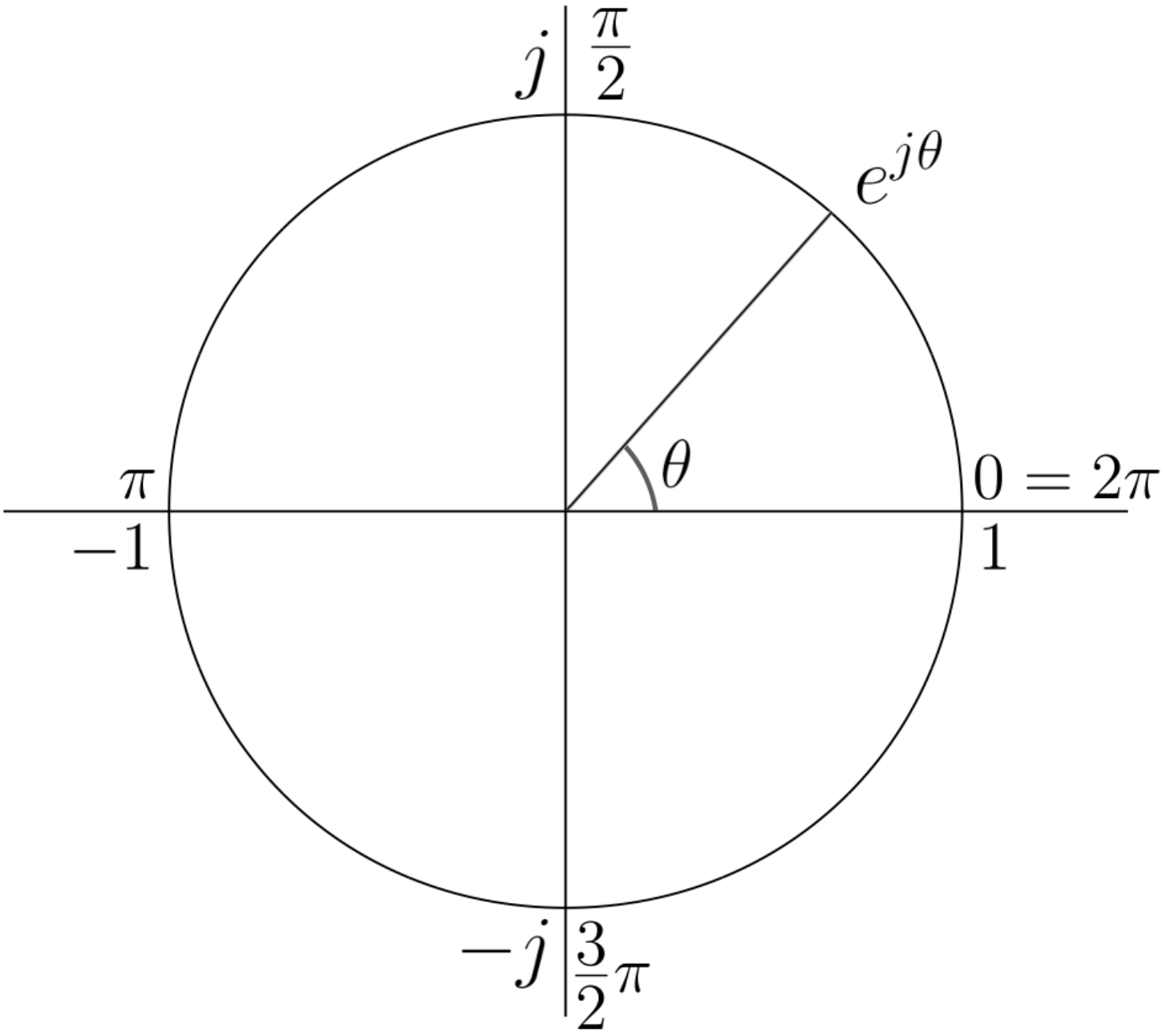
$$(a_1+jb_1)(a_2+jb_2)=(a_1a_2-b_1b_2)+j(a_1b_2+a_2b_1)$$

$$r_1r_2(\cos(\theta_1+\theta_2)+j\sin(\theta_1+\theta_2))$$

relación de Euler

$$e^{j\theta} = cos\theta + j \sin \theta$$

$$e \approx 2.71828182845904523536...$$


j
$\frac{\pi}{2}$
$e^{j\theta}$
θ
π
-1
$0 = 2\pi$
1
$-j$
$\frac{3}{2}\pi$

“la fórmula más hermosa de las matemáticas”

$$e^{j\pi} + 1 = 0$$

representaciones de un complejo

$$z = a + jb$$

$$z = r(\cos\theta + j\sin\theta)$$

$$z = re^{j\theta}$$

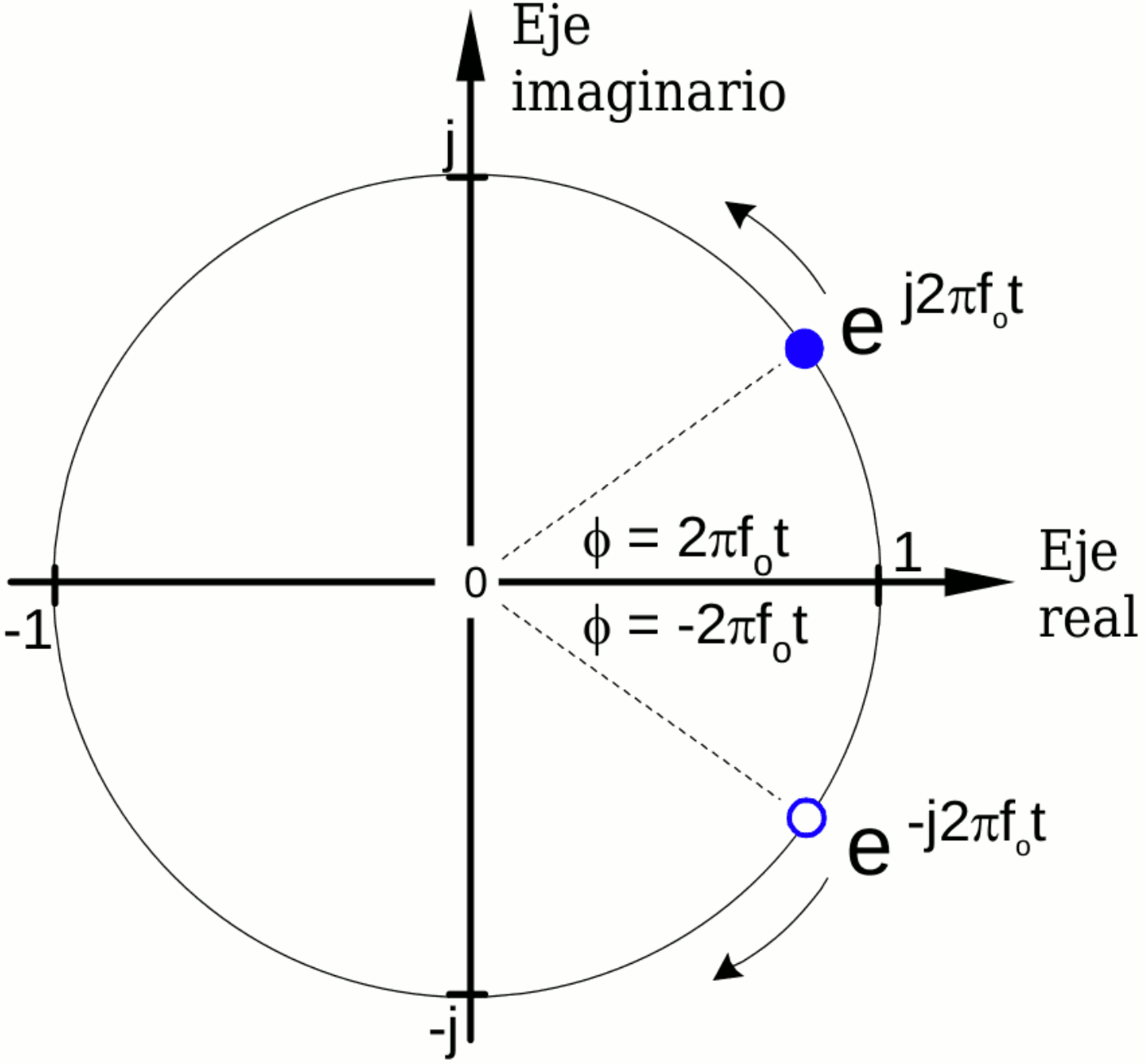
teorema de De Moivre

$$[\cos\theta + j\sin\theta]^n = \cos(n\theta) + j\sin(n\theta)$$

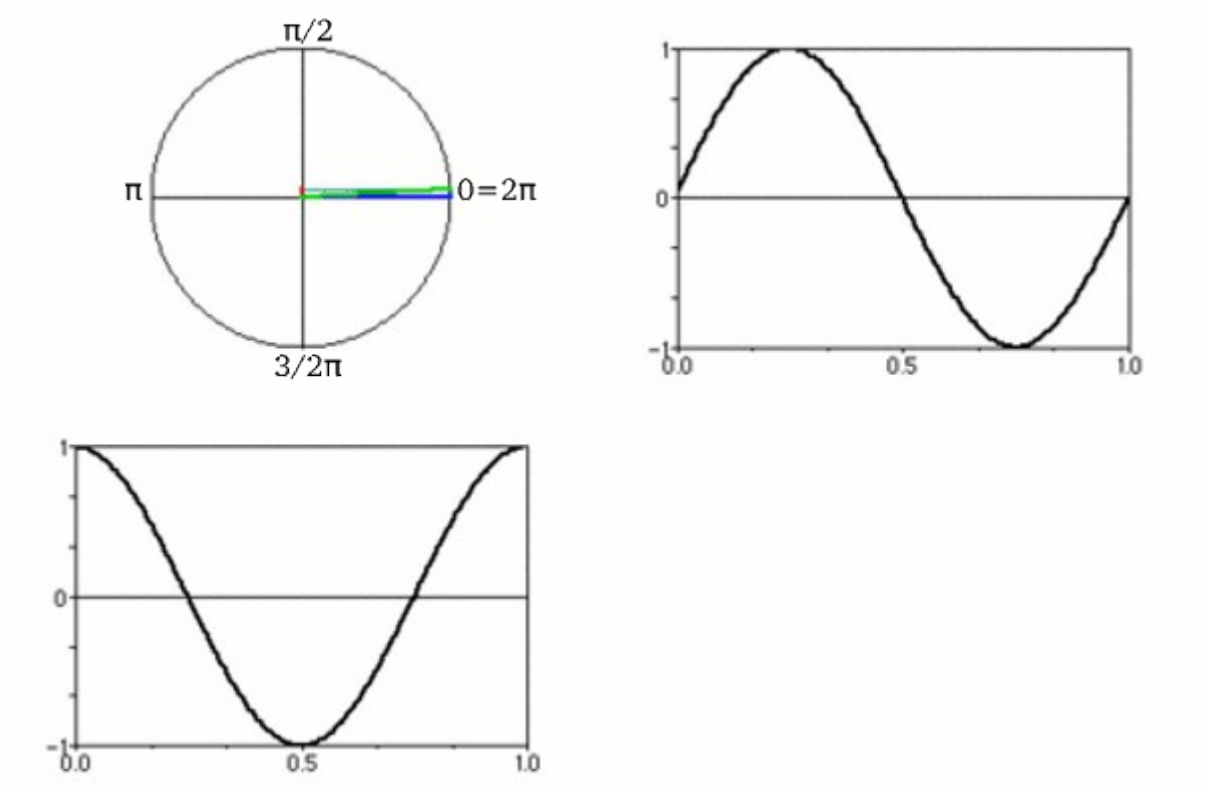
teorema de De Moivre

$$[\cos\theta + j\sin\theta]^n = \cos(n\theta) + j\sin(n\theta)$$

$$[e^{j\theta}]^n = e^{j\theta n} = \cos(n\theta) + j\sin(n\theta)$$


Eje
imaginario
j
$e^{j2\pi f_o t}$
$\phi = 2\pi f_o t$
1
Eje
real
-1
0
$\phi = -2\pi f_o t$
$e^{-j2\pi f_o t}$
-j

la sinusoide como fasor



seno complejo